# FEDERAL UNIVERSITY OF TECHNOLOGY MINNA

# APPLIED MATHEMATICS: THE REALITY, VALIDITY AND CONTEMPORARINESS OF MATHEMATICAL MODELING TO REAL LIFE PROBLEMS

***By***


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## 1.1 INTRODUCTION

Inaugural lectures have now taken the form of an informal meeting between the TOWN and the GOWN. Taking a very close look at this description and in relation with people's perceptions of certain disciplines it then becomes imperative at least with some not too friendly disciplines to make a shift from outright presentation of results from research to a milder hybrid of advocacy and result presentation. To this end, the introductory part of this short lecture is more of the advocacy of the ubiquitous property of Mathematics rather than being an occasion for a presentation of robust Mathematical proofs and theory.

Generally, it is a common but erroneous belief that mathematics has its jurisdiction limited to the classroom and that with the advent and advancement of high level computing it will die a natural death.

The first part of the assertion above can clearly not stand, as it will be seen in this presentation. The latter case may be explained off in the words of mathematicians that opine that most frequently encountered real life problems of note can all be reduced to mathematically solvable problems. Such problems undoubtedly require the understanding of the mathematics involved before the computer can ever be of use. In this lecture therefore, we are going to open closed gate of our consciousness to a few but remote areas of human endeavors where Mathematics plays some incontrovertible roles. The introductory part of this lecture is therefore a form of advocacy and enlightenment on the real life applications of Mathematics.

### Financial Mathematics and the Capital Market

At the center of modern global economic development and the basis of a huge industry is Mathematical Finance (Financial Mathematics) a child born as a consequence of the presentation of Louis Bachelier's doctorate thesis (theorie de la speculation). In his thesis, he introduced Brownian motion as a Mathematical model for stock prices years before the introduction of it by Einstein's classical theory. This is a continuous path Stochastic Process $(B(t_K), t \geq 0)$ in which increments $(B(t_k) - B(t_{k-1}))$, $(B(t_{k-1}) - B(t_{k-2}))$ are independent of the time $t_{k-2} \leq t_{k-1} \leq t_k$.

These increments distributed evenly with zero mean and variance with change in time. Years later, Black-Scholes model and several other models were put forward to advance the application of mathematics of finance building on the Brownian theories for stock pricing. These models estimated the volatility parameter from financial time series. Years later, the celebrated Black and Rubinstein(1979) option pricing model formula was introduced into financial market. This fundamental insight is the idea of perfect replication in which selling an option receives a premium acquire the obligation to deliver the exercise value at a time *T*. This is a derivative security in which the exercise value depends only on the movements on the underlying and is tradable.

*SimpleInterest*

The model for the computation of the *future value* $(FV)$ from a *present value* $(PV)$ with a simple interest *rate* $(R)$ for a period $(T)$ is given as;

$$FV = PV(1 + RT) \qquad (1)$$

Conversely, the present value that may be deposited to give an expected future value under the same interest regime maybe computed from (1) as;

$$PV = FV(1 + RT)^{-1} \qquad (2)$$

*CompoundInterest*

$$FV = PV\left(1 + \frac{R}{f}\right)^{T \times f} \qquad (3)$$

$f$ here is the frequency of computation per period.

## Mathematics in Decision Making (Operations Research)

A very versatile branch of mathematics with enormous application in a wide range of human activities is the science of decision making. It encompasses a wide range of problem solving techniques and methods applied in the pursuit of improved and efficient decision-making process. This includes among others; queuing theory, Stochastic processes and econometric methods, data development analysis neutral network etc.

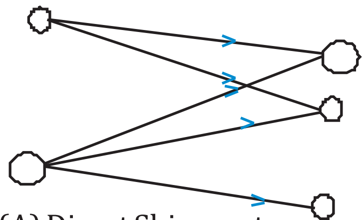
Some of the major sub disciplines in modern operation research include among others:

- Revenue management
- Transportation system.
- Computing and information technologies

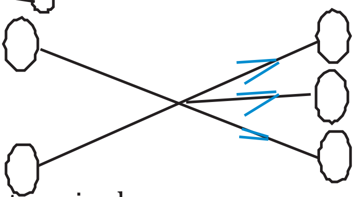
## A Simple Transportation Problem

The following sketches give an illustration of the transportation problem of a company whose desire is to transport its products from its warehouse to (a) designated outlet(s).

- direct shipping (A) and shipping through a distribution Centre (B)



(A) Direct Shipment



*m x n* links

(B) Shipment via a terminal

## Mathematics in Cybernetics

Cybernetics among various other definitions can be said to be a branch of mathematics that deals with problems of controls recursiveness and information, focuses in forms and patterns that connect. This also concerns itself with the study of system of any nature capable of processing, storing and retrieving of information meant for use in control. The word cybernetics was used in the context of study of self-governance by Plato to signify the governance of people. The physicist Andre-Mane Anyere in 1834 used this to denote the science of government in his classification system of human knowledge. Contemporary, cybernetics however began as an interdisciplinary study combining the fields of control systems, electrical network theory, mechanical engineering and logic modeling, evolutionary biology and neuroscience in the 1940s.

The early applications of system feedback in electronic circuit included control of gun mouths and radar antenna during the Second World War. This discipline was firmly established by some great scientist and mathematics such as Wiener, Rose Ahby, Alan Turina and Grey Walter. In particular, the invention of the Von Neumann cellular automaton is a logical following, which resulted in the concept whose same properties of genetic reproduction applied to social memes. Living cells and computer viruses are properties of the surprising universality of cybernetic study.

This study area has very versatile applications in:

- Artificial intelligence
- Robotics
- Control system
- Bio - Engineering
- Bio -Cybernetics
- Synthetic biology

- Neuroscience
- Psychology
- Sociology, etc.

**Mathematics in Music**

Mathematical theories are often used by rhetorical musicians to understand music even though it might be argued that music has no axiomatic foundation in mathematics, nonetheless, the basis often found exhibit a remarkable array of number properties because nature is itself amazingly Mathematical. Several attempts to structure and communicate new ways of composing and hearing music have led to musical applications of set theory, abstract algebra and number theory. Many composers indeed have incorporated the golden ratio and Fibonacci numbers into their work.

Elementary Mathematical Set theory is often used as a tool in musical set theory to organize musical objects aid in the description of their relationships. Operations such as transposition and inversion called isometrics because of their representation properties to maintain intervals between tones are used in analyzing certain structures in musical set theory.

Similarly, certain theorems in abstract algebra have proved to be versatile in the expansion of some methods in musical set theory, for instance the pitch classes in a uniformly tempered octave form an *abelian group* with 12 elements. It is therefore possible to describe such intonation in terms of the same *abelian group*. The chromatic scale has a free and transitive action defined as notes which therefore may be viewed as a fossil of the group 21/124. Finally, elements of Real and Complex Analysis are known to have been applied in certain musical set theory. In particular, the theory of *Riemann Zeta function* has been used extensively in the study of equal division of the octave.

## Mathematics in Cryptography

Modern cryptography is a discipline that is heavily based on Mathematical theory. Several algorithms in cryptography are formulated assumptions that are intended to make such algorithms difficult if not impossible to break by any adversary. It actually may be possible a task theoretically to break such a system but practically impossible to do so by any known method.

These schemes therefore are said to be computationally secured. For instance, theories such as the improvement in integer factorization algorithms and faster computing technology all require that these solutions be continually adapted. Indeed, there exist information theoretically secure schemes which probably cannot be broken into even with an unlimited high level computing power. However, such schemes unfortunately may be practically insecure. This is the practice and study of techniques for granting security of communication in the presence of an undesirable third party known in this context as adversary. This can more generally be viewed as a system that constrains and to block adversaries' access to various aspects of information attributes of data confidentiality, data integrity, authenticity and non-reputation, which are all central to modern cryptography. Very common and familiar real life application of cryptography include ATM cards, computer passwords and electronic commerce.

Prior to modern day developed electronic encryption, cryptography was based primitively on such practice of encrypting plaintext to cipher text by a sender in which the recipient had to decrypt the cipher text to plaintext. These are much more difficult to implement than the best-known theatrically breakable but automatically secure ones.

**Linear Programming (PRODUCTION)**

Linear programming as with many other mathematical techniques was developed primarily for the purpose of solving certain class of practical problems. During the Second World War, several problems arose in connection with the optimal deployment of aircraft, submariner, men materials and transport. Similar problems also arose in the post-war era in industries and government like economic problems, manufacturing, optimal choice of shipping routes etc. This class of problems is best described as the "optimal allocation of scarce resources, classical mathematical methods of calculus and algebra failed in solving these problems for the fact that the formulations of these problems are basically inequalities rather than equations. In majority of this class of problem, every variable appears linearly both in the inequality constraints (scarceness of resources) and in the objective function (quantity to be optimized). This formulation and solution of this class of problem is therefore the subject of linear inequality.

A typical application of Linear Programming Problems (LPP) is in a manufacturing company that produces different products $X_1, X_2, X_3, X_4, \ldots\ldots\ldots, X_{m-1}, X_m$ from $m$ production variables (raw materials) $a_1, a_2, a_1, a_2, \ldots\ldots\ldots, a_{n-1}, a_n$. It will also be assumed the $j$th product that is, $X_j$ requires $a_{ij}$ quantity of the $i$th production variable (resource) per unit. It is finally assumed that the resources for the production are grouped by some availability constraints given that there is available to the company $b_j$ quantity of the $j$th raw material.

In term of the final product(out-put), it is assumed that per unit profit (overhead) contribution of production $p$ is given as $c_p$ This therefore gives the total profit for the production of the $m$ products $X_1, X_2, X_3, X_4, \ldots\ldots\ldots, X_{m-1}, X_m$ as

$$f = c_1X_1 + c_2X_2 + c_3X_3 + \ldots\ldots + c_mX_m \qquad (1)$$

This expression above is the so-called objective function and in this case is to be maximized (1).

Subject to the following inequality constraints:

$$\left.\begin{array}{l} a_{11}X_1 + a_{12}X_2 + a_{13}X_3 + \ldots\ldots + a_{1m}X_m \le b_1 \\ a_{21}X_1 + a_{22}X_2 + a_{23}X_3 + \ldots\ldots + a_{2m}X_m \le b_2 \\ a_{31}X_1 + a_{32}X_2 + a_{33}X_3 + \ldots\ldots + a_{3m}X_m \le b_3 \\ . \\ a_{n1}X_1 + a_{n2}X_2 + a_{n3}X_3 + \ldots\ldots + a_{nm}X_m \le b_n \end{array}\right\} \qquad (2)$$

The inequality in (2) is the inequality constraints and the non-negativity restriction.

The LPP is therefore to maximize (minimize) (1) subject to the inequalities in (2).

**Mathematical Modeling of Real Life Problem**

This is the process of transforming real life problems into mathematical concepts and languages. A wide variety of real life problems in the natural sciences, engineering disciplines, social sciences, physical sciences etc. can be modeled into mathematically solvable problems, the results of which must necessarily conform to natural laws and real life observation. In this wise, it is therefore commonplace for physicists, engineers, statisticians, operation research analysts and economics to apply extensively the use of mathematical models. These models may take the form but not limited to dynamic models, differential equations, integral equations, integro-differential equations or any combination of these as theoretical models. A particular problem therefore may involve the use of the above mentioned or the combination.

It is however a known fact that the quality of the results of any scientific research depends very much on how well the mathematical models of functions that establish these relationships are formulated. In the fields of business and engineering, mathematical models may be applied as an optimization tools (maximize or in maximize). Of course, the system that relate input to output on the other also depend on certain variables such as decision variables, state variables, exogenous variables (parameters or constants) and random variables that are often dependent on the decision, input, random and exogenous variables, which the output variable are dependent on the state of the system. Functions of the output variables or state variables are often used to represent the objectives and constraints of the system. Are its users developed on the theoretic platform with standard experimental results? Of course, the lack of agreement of the theoretical mathematical measurements always leads researchers to important advances as this invariably brings about new theories.

The complexity of a mathematical model most often determines the quality of its results and conformity with natural expectations. Since mathematical models are formulations from real life observations and diligent experimental processes, a good mathematical model must agree to a very large extent to natural predictions.

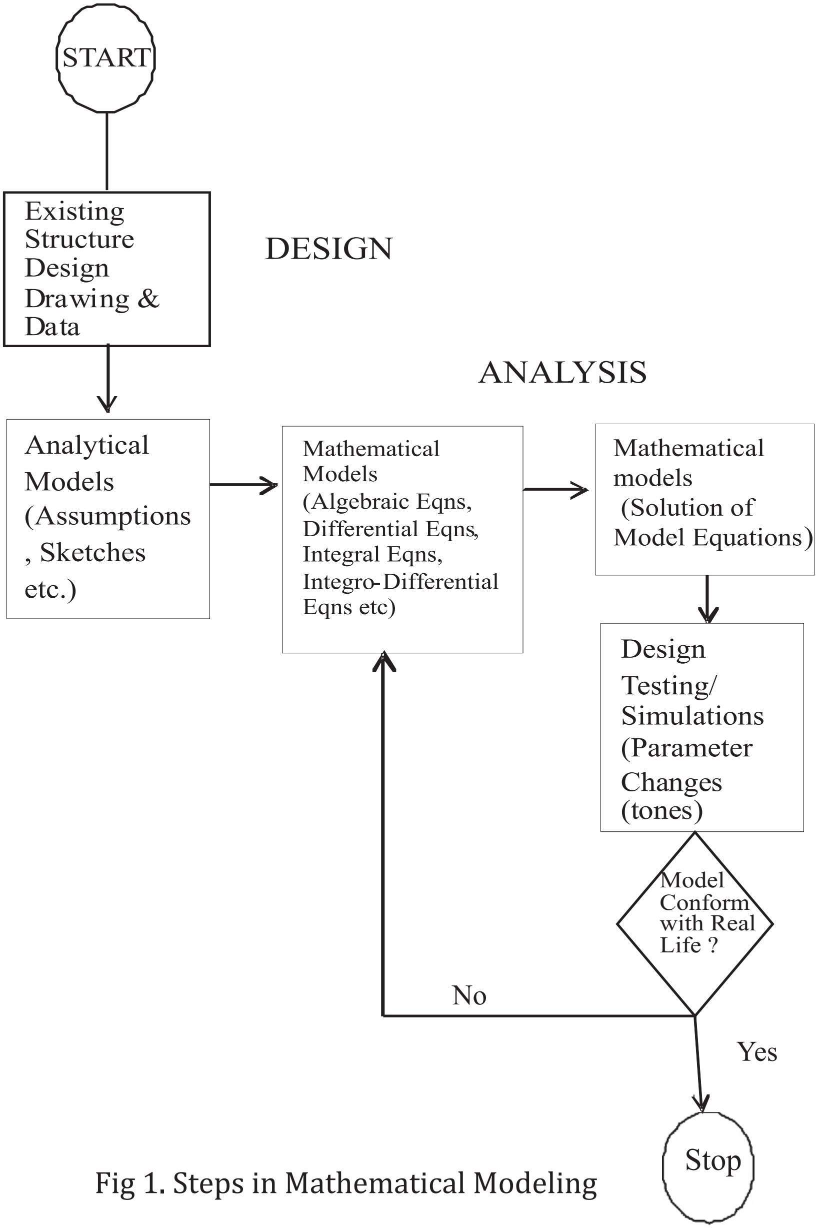



Fig 1. Steps in Mathematical Modeling

**Real Life Application to Dynamics: My Contribution.**

The study of dynamics in the context of Applied Mathematics may be classified in two broad areas:

- Dynamics of Structure (solid state) and
- Dynamics of Fluid (Liquid, Gas & Plasma)

This lecture therefore is to present a synopsis of my contribution to scholarship, which is domiciled in the area of application of Mathematics to structural dynamics with specific attention to Plate – Moving Load problems and Computational Fluid Dynamics with main focus in the area of

Magneto hydro dynamic flow of Non Newtonian Fluid.

**Resonance**

To every dynamic structure, there are specific frequencies of vibration referred to as the natural frequencies of vibration of the structure. The phenomenon of resonance is not limited to the field of structural dynamics but in other fields such as electronics, acoustics, atomic/nuclear physics etc. This phenomenon has an interesting ambivalent characteristic as it actually may be desirable in certain fields but destructive in some others. For instance, radio communication system is entirely dependent on the principle of resonance for a successful transmission as the frequency of the receiver (radio transistor) set into resonance with the incoming radio wave for a successful broadcast. On the contrary, in the fields of structural and highway engineering this constitutes a nuisance as it causes destruction of structures. Bridges and other structures are known to have collapsed as results of resonance occurring between these structures and certain signals traversing them. A handy example of engineering structure that was affected by resonance as a result of external excitation is the Tacoma Narrows Bridge.

Plate I: Tacoma Narrows Bridge (Source: UW libraries)

## 1.1 **The Dynamic Analysis of an Elastic Plate**

Plates are widely used structures with wide engineering applications in aircrafts, nuclear vessels, hydraulics, bridges and roads. There have been a great deal of research on the analysis of structures (shells, plates and beams) with consideration for various factors such as displacements, thickness variation, stresses, curvature, effect of surrounding media, loads and masses. In particular, the problems of moving masses and forces over plates and beams have been a subject of investigation in Mathematics, Physics and Engineering because of their extensive use in everyday life.

### 1.1.1 Moving load on a thin plate

We consider the problem on a thin plate characterized by;

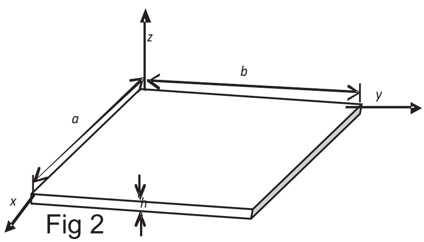



Fig 2

The uniform elastic plate of dimension *a* by *b* mass *m* with simple ends resting on a viscoelastic foundation of coefficient $\varepsilon_0$. Following Awodola & Omolofe (2018) the governing differential equation for the plate is therefore given by;

$$D\left(\frac{\partial^4 u}{\partial x^4}+\frac{\partial^4 u}{\partial x^2 \partial y^2}+\frac{\partial^4 u}{\partial y^4}\right)-N_x \frac{\partial^2 u}{\partial x^2}-N_y \frac{\partial^2 u}{\partial y^2}+\rho \frac{\partial^2 u}{\partial t^2}+\varepsilon_0 \frac{\partial u}{\partial t}+\kappa u$$

$$=F(x,y,t)\ ;\quad 0 \le x \le a, 0 \le y \le b;\ h \le {}^{1}\!/_{9}\ \min(a,b) \qquad (4)$$

The parameters retain their familiar definitions.

$$F(x,y,t)=P\cos\omega t \cdot \delta[x-(x_0+R\cos\beta t)\cdot\delta(y-(y_0+R\sin\beta t))] \qquad (5)$$

In particular a simply –supported plate satisfies the boundary conditions $[u, u'']_B=0$ (6)

The subscript *B* above implies values taken at the boundary of the plate, hence.

$$u(x,y\ t)=\lambda^{-1}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left\{\left[\left[\begin{matrix} J_0(R_b)J_0(R_a)X_1+\sum_{r=1}^{\infty}J_{2r}(R_b)J_0(R_a)X_2 \\ +\sum_{p=1}^{\infty}(-1)^p J_{2p}(R_b)J_0(R_a)X_3+\sum_{r=1}^{\infty}\sum_{p=1}^{\infty}J_{2r}(R_b)J_{2p}(R_a)X_4 \end{matrix}\right] SinASinB+\right]\right.$$

$$+\sum_{r=1}^{\infty}J_{2r}(R_b)J_0(R_a)X_5 SinACosB+\left[\begin{matrix}\sum_{p=1}^{\infty}(-1)^p J_0(R_b)J_{2p+1}(R_a)X_6+ \\ +\sum_{r=1}^{\infty}\sum_{p=1}^{\infty}(-1)^r J_{2r}(R_b)J_{2p}(R_a)X_7\end{matrix}\right]CosASinB$$

$$\left.+\sum_{r=1}^{\infty}\sum_{p=1}^{\infty}(-1)^r J_{2r+1}(R_b)J_{2p+1}(R_a)X_8 CosACosB\right\} Sin\left(\frac{m\pi x}{a}\right)Sin\left(\frac{n\pi y}{b}\right)$$

where

$$X_1=\frac{\gamma(\sin(\lambda-a_0)t-e^{-\gamma t}\sin\lambda t)+a_0(\cos(\lambda-a_0)t-a_0^* e^{-\gamma t}\cos\lambda t)}{a_0^2+\gamma^2}$$

$$+\frac{\gamma(\sin(\lambda-a_1)t-e^{-\gamma t}\sin\lambda t)+a_1(\cos(\lambda-a_1)t-a_1^* e^{-\gamma t}\cos\lambda t)}{a_1^2+\gamma^2}$$

$$X_2 = \frac{\gamma(\cos(\lambda - a_6)t - e^{-\gamma t}\cos\lambda t) + a_6(\sin(\lambda - a_6)t - a_6^* e^{-\gamma t}\sin\lambda t)}{a_6^2 + \gamma^2}$$

$$- \frac{\gamma(\cos(\lambda - a_7)t - e^{-\gamma t}\cos\lambda t) + a_7(\sin(\lambda - a_7)t - a_7^* e^{-\gamma t}\sin\lambda t)}{a_7^2 + \gamma^2}$$

$$- \frac{\gamma(\cos(\lambda - a_9)t - e^{-\gamma t}\cos\lambda t) + a_9(\sin(\lambda - a_9)t - a_9^* e^{-\gamma t}\sin\lambda t)}{a_9^2 + \gamma^2}$$

$$+ \frac{\gamma(\cos(\lambda - a_8)t - e^{-\gamma t}\cos\lambda t) + a_6(\sin(\lambda - a_8)t - a_8^* e^{-\gamma t}\sin\lambda t)}{a_8^2 + \gamma^2}$$

$$X_3 = \frac{\gamma(\sin(\lambda - a_2)t - e^{-\gamma t}\sin\lambda t) - a_2(\cos(\lambda - a_2)t - a_2^* e^{-\gamma t}\cos\lambda t)}{a_2^2 + \gamma^2}$$

$$+ \frac{\gamma(\sin(\lambda - a_3)t - e^{-\gamma t}\sin\lambda t) - a_3(\cos(\lambda - a_3)t - a_3^* e^{-\gamma t}\cos\lambda t)}{a_3^2 + \gamma^2}$$

$$+ \frac{\gamma(\sin(\lambda - a_4)t - e^{-\gamma t}\sin\lambda t) - a_4(\cos(\lambda - a_4)t - a_4^* e^{-\gamma t}\cos\lambda t)}{a_4^2 + \gamma^2}$$

$$+ \frac{\gamma(\sin(\lambda - a_5)t - e^{-\gamma t}\sin\lambda t) - a_5(\cos(\lambda - a_5)t - a_5^* e^{-\gamma t}\cos\lambda t)}{a_5^2 + \gamma^2}$$

$$X_4 = \frac{\gamma(\cos(\lambda - a_{18})t - e^{-\gamma t}\cos\lambda t) - a_{18}(\sin(\lambda - a_{18})t - a_{18}^* e^{-\gamma t}\sin\lambda t)}{a_{18}^2 + \gamma^2}$$

$$+ \frac{\gamma(\cos(\lambda - a_{20})t - e^{-\gamma t}\cos\lambda t) - a_{20}(\sin(\lambda - a_{20})t - a_{20}^* e^{-\gamma t}\sin\lambda t)}{a_{20}^2 + \gamma^2}$$

$$- \frac{\gamma(\cos(\lambda - a_{21})t - e^{-\gamma t}\cos\lambda t) - a_{18}(\sin(\lambda - a_{21})t - a_{21}^* e^{-\gamma t}\sin\lambda t)}{a_{21}^2 + \gamma^2}$$

$$- \frac{\gamma(\cos(\lambda - a_{19})t - e^{-\gamma t}\cos\lambda t) - a_{19}(\sin(\lambda - a_{19})t - a_{19}^* e^{-\gamma t}\sin\lambda t)}{a_{19}^2 + \gamma^2}$$

$$- \frac{\gamma(\cos(\lambda - a_{23})t - e^{-\gamma t}\cos\lambda t) - a_{18}(\sin(\lambda - a_{23})t - a_{23}^* e^{-\gamma t}\sin\lambda t)}{a_{23}^2 + \gamma^2}$$

$$-\frac{\gamma(\cos(\lambda-a_{24})t-e^{-\gamma t}\cos\lambda t)-a_{24}(\sin(\lambda-a_{22})t-a_{24}^{*}e^{-\gamma t}\sin\lambda t)}{a_{24}^{2}+\gamma^{2}}$$

$$+\frac{\gamma(\cos(\lambda-a_{22})t-e^{-\gamma t}\cos\lambda t)-a_{22}(\sin(\lambda-a_{22})t-a_{22}^{*}e^{-\gamma t}\sin\lambda t)}{a_{22}^{2}+\gamma^{2}}$$

$$+\frac{\gamma(\cos(\lambda-a_{25})t-e^{-\gamma t}\cos\lambda t)-a_{25}(\sin(\lambda-a_{18})t-a_{25}^{*}e^{-\gamma t}\sin\lambda t)}{a_{25}^{2}+\gamma^{2}}$$

$$X_5=\frac{\gamma(\cos(\lambda-a_{14})t-e^{-\gamma t}\cos\lambda t)-a_{14}(\sin(\lambda-a_{14})t-a_{14}^{*}e^{-\gamma t}\sin\lambda t)}{a_{14}^{2}+\gamma^{2}}$$

$$+\frac{\gamma(\cos(\lambda-a_{29})t-e^{-\gamma t}\cos\lambda t)-a_{18}(\sin(\lambda-a_{29})t-a_{29}^{*}e^{-\gamma t}\sin\lambda t)}{a_{29}^{2}+\gamma^{2}}$$

$$-\frac{\gamma(\cos(\lambda-a_{27})t-e^{-\gamma t}\cos\lambda t)-a_{27}(\sin(\lambda-a_{27})t-a_{27}^{*}e^{-\gamma t}\sin\lambda t)}{a_{27}^{2}+\gamma^{2}}$$

$$-\frac{\gamma(\cos(\lambda-a_{28})t-e^{-\gamma t}\cos\lambda t)-a_{28}(\sin(\lambda-a_{28})t-a_{28}^{*}e^{-\gamma t}\sin\lambda t)}{a_{28}^{2}+\gamma^{2}}$$

$$-\frac{\gamma(\cos(\lambda-a_{33})t-e^{-\gamma t}\cos\lambda t)-a_{33}(\sin(\lambda-a_{33})t-a_{33}^{*}e^{-\gamma t}\sin\lambda t)}{a_{33}^{2}+\gamma^{2}}$$

$$-\frac{\gamma(\cos(\lambda-a_{34})t-e^{-\gamma t}\cos\lambda t)-a_{34}(\sin(\lambda-a_{34})t-a_{34}^{*}e^{-\gamma t}\sin\lambda t)}{a_{34}^{2}+\gamma^{2}}$$

$$+\frac{\gamma(\cos(\lambda-a_{32})t-e^{-\gamma t}\cos\lambda t)-a_{32}(\sin(\lambda-a_{32})t-a_{32}^{*}e^{-\gamma t}\sin\lambda t)}{a_{32}^{2}+\gamma^{2}}$$

$$+\frac{\gamma(\cos(\lambda-a_{35})t-e^{-\gamma t}\cos\lambda t)-a_{35}(\sin(\lambda-a_{35})t-a_{35}^{*}e^{-\gamma t}\sin\lambda t)}{a_{35}^{2}+\gamma^{2}}$$

$$X_6=\frac{\gamma(\sin(\lambda-a_{10})t-e^{-\gamma t}\cos\lambda t)-a_{18}(\cos(\lambda-a_{18})t-a_{18}^{*}e^{-\gamma t}\cos\lambda t)}{a_{10}^{2}+\gamma^{2}}$$

$$+\frac{\gamma(\sin(\lambda-a_{11})t-e^{-\gamma t}\sin\lambda t)-a_{18}(\cos(\lambda-a_{11})t-a_{11}^{*}e^{-\gamma t}\cos\lambda t)}{a_{11}^{2}+\gamma^{2}}$$

$$+\frac{\gamma(\sin(\lambda-a_{12})t-e^{-\gamma t}\sin\lambda t)-a_{18}(\cos(\lambda-a_{12})t-a_{12}^{*}e^{-\gamma t}\cos\lambda t)}{a_{12}^{2}+\gamma^{2}}$$

$$+\frac{\gamma(\sin(\lambda-a_{13})t-e^{-\gamma t}\sin\lambda t)-a_{13}(\cos(\lambda-a_{13})t-a_{13}^{*}e^{-\gamma t}\cos\lambda t)}{a_{13}^{2}+\gamma^{2}}$$

$$X_{7}=\frac{\gamma(\cos(\lambda-a_{26})t-e^{-\gamma t}\cos\lambda t)-a_{26}(\sin(\lambda-a_{13})t-a_{26}^{*}e^{-\gamma t}\sin\lambda t)}{a_{26}^{2}+\gamma^{2}}$$

$$+\frac{\gamma(\cos(\lambda-a_{29})t-e^{-\gamma t}\cos\lambda t)-a_{29}(\sin(\lambda-a_{13})t-a_{29}^{*}e^{-\gamma t}\sin\lambda t)}{a_{26}^{2}+\gamma^{2}}$$

$$-\frac{\gamma(\cos(\lambda-a_{27})t-e^{-\gamma t}\cos\lambda t)-a_{27}(\sin(\lambda-a_{27})t-a_{27}^{*}e^{-\gamma t}\sin\lambda t)}{a_{27}^{2}+\gamma^{2}}$$

$$-\frac{\gamma(\cos(\lambda-a_{28})t-e^{-\gamma t}\cos\lambda t)-a_{28}(\sin(\lambda-a_{28})t-a_{28}^{*}e^{-\gamma t}\sin\lambda t)}{a_{28}^{2}+\gamma^{2}}$$

$$-\frac{\gamma(\cos(\lambda-a_{33})t-e^{-\gamma t}\cos\lambda t)-a_{33}(\sin(\lambda-a_{33})t-a_{33}^{*}e^{-\gamma t}\sin\lambda t)}{a_{33}^{2}+\gamma^{2}}$$

$$-\frac{\gamma(\cos(\lambda-a_{34})t-e^{-\gamma t}\cos\lambda t)-a_{34}(\sin(\lambda-a_{34})t-a_{34}^{*}e^{-\gamma t}\sin\lambda t)}{a_{34}^{2}+\gamma^{2}}$$

$$+\frac{\gamma(\cos(\lambda-a_{33})t-e^{-\gamma t}\cos\lambda t)-a_{28}(\sin(\lambda-a_{33})t-a_{33}^{*}e^{-\gamma t}\sin\lambda t)}{a_{33}^{2}+\gamma^{2}}$$

$$+\frac{\gamma(\cos(\lambda-a_{35})t-e^{-\gamma t}\cos\lambda t)-a_{35}(\sin(\lambda-a_{35})t-a_{35}^{*}e^{-\gamma t}\sin\lambda t)}{a_{35}^{2}+\gamma^{2}}$$

$$X_{8}=\frac{\gamma(\cos(\lambda-a_{21})t-e^{-\gamma t}\cos\lambda t)-a_{21}(\sin(\lambda-a_{21})t-a_{21}^{*}e^{-\gamma t}\sin\lambda t)}{a_{21}^{2}+\gamma^{2}}$$

$$-\frac{\gamma(\cos(\lambda-a_{24})t-e^{-\gamma t}\cos\lambda t)-a_{24}(\sin(\lambda-a_{24})t-a_{24}^{*}e^{-\gamma t}\sin\lambda t)}{a_{24}^{2}+\gamma^{2}}$$

$$-\frac{\gamma(\cos(\lambda-a_{20})t-e^{-\gamma t}\cos\lambda t)-a_{20}(\sin(\lambda-a_{20})t-a_{20}^{*}e^{-\gamma t}\sin\lambda t)}{a_{20}^{2}+\gamma^{2}}$$

$$-\frac{\gamma(\cos(\lambda-a_{25})t-e^{-\gamma t}\cos\lambda t)-a_{25}(\sin(\lambda-a_{25})t-a_{25}^{*}e^{-\gamma t}\sin\lambda t)}{a_{25}^{2}+\gamma^{2}}$$

$$-\frac{\gamma(\cos(\lambda-a_{39})t-e^{-\gamma t}\cos\lambda t)-a_{39}(\sin(\lambda-a_{39})t-a_{39}^{*}e^{-\gamma t}\sin\lambda t)}{a_{39}^{2}+\gamma^{2}}$$

$$+\frac{\gamma(\cos(\lambda-a_{41})t-e^{-\gamma t}\cos\lambda t)-a_{41}(\sin(\lambda-a_{41})t-a_{41}^{*}e^{-\gamma t}\sin\lambda t)}{a_{41}^{2}+\gamma^{2}}$$

$$+\frac{\gamma(\cos(\lambda-a_{38})t-e^{-\gamma t}\cos\lambda t)-a_{38}(\sin(\lambda-a_{38})t-a_{38}^{*}e^{-\gamma t}\sin)\lambda t}{a_{38}^{2}+\gamma^{2}}$$

$$+\frac{\gamma(\cos(\lambda-a_{40})t-e^{-\gamma t}\cos\lambda t)-a_{40}(\sin(\lambda-a_{40})t-a_{40}^{*}e^{-\gamma t}\sin\lambda t)}{a_{40}^{2}+\gamma^{2}}$$

**Resonance**

From the deflection profile of the vibrating plate as given above we thus have the following as the conditions of the resonance of the plate- moving load problem.

$$\lambda=2m\beta,\ \lambda+\omega=2m\beta,\ \lambda+\omega=(2m+1)\beta$$

$$|\lambda-\omega|=2m\beta$$

$$|\lambda-\omega|=2(m+n+1)\beta$$

$$|\lambda-\omega|=2(m-n+1)\beta$$

$$|\lambda-\omega|=2(2m-2n+1)\beta$$

$$|\lambda-\omega|=2(2m-2n-1)\beta,\quad m,n\in\mathbb{N}$$

These are the established conditions under which resonance of the structure will occur and hence he designers of such structures can predetermine allowable frequency $\omega$ and $\beta$ knowing fully in advance the natural frequency $\lambda$ of the structure to for stall the occurrence of resonance.

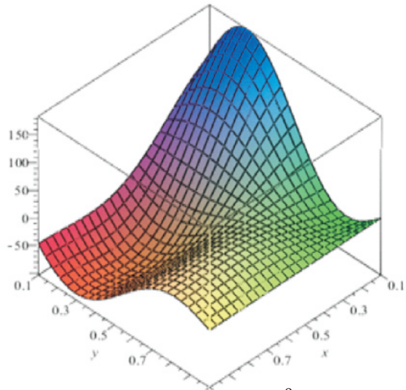



FIG 3: $\kappa = 10^{-9}$

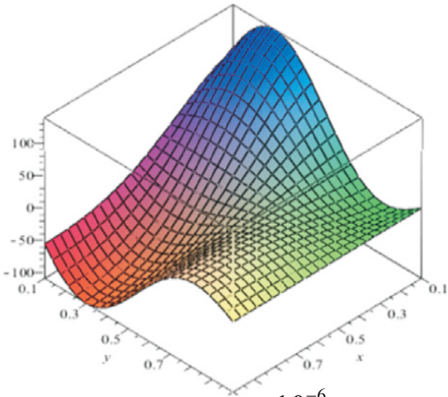



FIG 4: $\kappa = 10^{-6}$

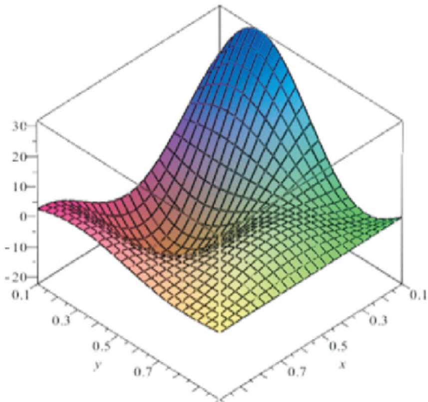



FIG 5: $\kappa = 10^{0}$

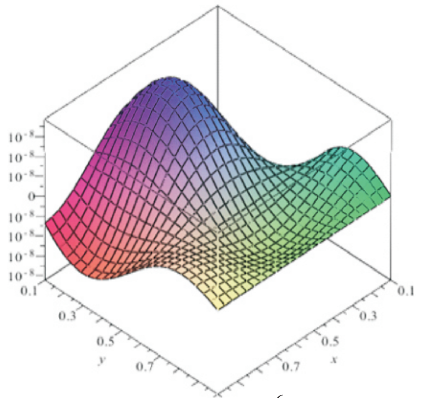



FIG 6: $\kappa = 10^{6}$

*Dimensionless Amplitudes of the Plate with Various Values of* $\kappa$ *(coefficient of viscosity/ Foundation Parameter)*

## 1.2 **Problem of Moving Load on a Thick Non Uniform (Anisotropic) Plate**

Several authors have in particular investigated the effect of shear deformations and rotary inertia on plates and beams including Sathymoorthy and Chia (1980), Koo (2014) mostly with focus on isotropic plates. There has been very little focus on anisotropic plates and in particular, the effect of shear deformations and rotary inertia, with varying loads traversing the plates. According to Toorani & Lakis (2000), transverse shear deformation plays a very important role in reducing the effective flexural stiffness of anisotropic laminated plates and shells because their in-plane elastic modulus to transverse shear modulus ratio is high.

The effect of moving loads and masses on isotropic plates and beams have also been studied by authors including Aiyesimi (1999b, 2000, 2004, 2011), Oni & Awodola (2011), Esen (2015) and have given solutions using analytic and approximate methods such as the finite difference, Galerkin, Rayleigh-Ritz, transfer matrix and finite element methods but none on anisotropic plates. In order to fill the void of studies on moving mass on anisotropic plates, the research tends to investigate the effects of shear deformation and rotary inertia on anisotropic plates traversed by varying moving masses.

### 1.2.1Problem Formulation

Our model equation is an extension and modification of the general single equation of a plate which considered the influence of rotary inertia and shear on flexural motions of elastic plates as proposed by Mindlin (1951) and also the introduction of the Vlasov and Leont'ev (1966) bi-parametric foundation. We shall presently investigate the response of the plate to two types of loads namely;

I. Moving **FORCE** ii. Moving **Mass**

### 1.2.2 Moving Force and Moving Mass

This is adopted from the moving force $P(\xi, \eta, \iota)$ model of Aiyesimi (2009a) together with the convective acceleration (inertia effect) as defined in Aiyesimi (2009*b*)

Thus, our model governing equation is given as

$$\left.\begin{aligned}&\left[\nabla^2 D_d(\xi,\eta)-\left(\frac{\mu_d(\xi,\eta)D_d(\xi,\eta)}{hG_d}+R_0\right)\frac{\partial^2}{\partial\iota^2}\right]\nabla^2 U(\xi,\eta,\iota)+\frac{\mu_d(\xi,\eta)R_0}{hG_d}\frac{\partial^4 U(\xi,\eta,\iota)}{\partial\iota^4}\\&+\mu_d(\xi,\eta)\frac{\partial^2 U(\xi,\eta,\iota)}{\partial\iota^2}+\left(k-G_s\nabla^2\right)U(\xi,\eta,\iota)=Mg\delta(\xi-v_\xi\iota)\delta(\eta-v_\eta\iota)\\&-M\delta(\xi-v_\xi\iota)\delta(\eta-v_\eta\iota)\frac{1}{g}\left[\frac{\partial^2}{\partial\iota^2}+\left(2V_d\frac{\partial}{\partial\iota}+a_d\right)\left(\frac{\partial}{\partial\xi}+\frac{\partial}{\partial\eta}\right)+V_d^2\nabla^2\right]U(\xi,\eta,\iota)\end{aligned}\right\}\quad(12)$$

$$\left.\begin{aligned}D_d(\xi,\eta)&=D_0\left(1-\frac{2\xi}{a}+\frac{2\xi^2}{a^2}\right)\left(1-\frac{2\eta}{b}+\frac{2\eta^2}{b^2}\right)\\\mu_d(\xi,\eta)&=\mu_0\left(1-\frac{2\xi}{a}+\frac{2\xi^2}{a^2}\right)\left(1-\frac{2\eta}{b}+\frac{2\eta^2}{b^2}\right)\end{aligned}\right\}\quad(13)$$

$$\left.\begin{aligned}&U(x,y,t)\big|_{t=0}=U_0;\quad U_t(x,y,t)\big|_{t=0}=0\\&U_{tt}(x,y,t)\big|_{t=0}=0;\quad U_{ttt}(x,y,t)\big|_{t=0}=0\end{aligned}\right\}\quad(14)$$

$$\varepsilon_0=\frac{P_0}{\mu_0 D_3 I_{B11} I_{B12} g},\ \varphi_1^2=\frac{B_2}{B_1};\ \varphi_2^2=\frac{B_3}{B_1}\quad(15)$$

$$\begin{aligned}U(x,y,t)=&\sum_{n_i}^{\infty}\sum_{n_j}^{\infty}\frac{1}{\left(\varphi_{f1}^2-\varphi_{f2}^2\right)}\left\{\frac{P_f}{2}\left[\frac{1+A_{ni}A_{nj}}{\varphi_{f2}^2-\omega_1^2}\left(\cos\omega_1 t-\cos\varphi_{f2}t\right)\right.\right.\\&\left.-\frac{1-A_{ni}A_{nj}}{\varphi_{f2}^2-\omega_2^2}\left(\cos\omega_2 t-\cos\varphi_{f2}t\right)-\frac{1+A_{ni}A_{nj}}{\varphi_{f1}^2-\omega_1^2}\left(\cos\omega_1 t-\cos\varphi_{f1}t\right)-\frac{1-A_{ni}A_{nj}}{\varphi_{f1}^2-\omega_2^2}\left(\cos\omega_2 t-\cos\varphi_{f1}t\right)\right]\\&+\frac{1}{\varphi_{f2}}\left[\frac{A_{nj}-A_{ni}}{\varphi_{f2}^2-\omega_1^2}\left(\varphi_{f2}\sin\omega_1 t-\omega_1\sin\varphi_{f2}t\right)+\frac{A_{nj}+A_{ni}}{\varphi_{f2}^2-\omega_2^2}\left(\varphi_{f2}\sin\omega_2 t-\omega_2\sin\varphi_{f2}t\right)\right]\\&+\frac{1}{\varphi_{f1}}\left[\frac{A_{nj}-A_{ni}}{\varphi_{f1}^2-\omega_1^2}\left(\varphi_{f1}\sin\omega_1 t-\omega_1\sin\varphi_{f1}t\right)+\frac{A_{nj}+A_{ni}}{\varphi_{f1}^2-\omega_2^2}\left(\varphi_{f1}\sin\omega_2 t-\omega_2\sin\varphi_{f1}t\right)\right]\\&+\frac{2}{\varphi_{f2}\left[\left(\varphi_{f2}+\vartheta_x\right)^2+\vartheta_y^2\right]\left[\left(\varphi_{f2}-\vartheta_x\right)^2+\vartheta_y^2\right]}\left\{\varphi_{f2}\left(\varphi_{f2}^2-\omega_1\omega_2\right)\left(B_{nj}\sinh\vartheta_y t+C_{nj}\cosh\vartheta_y t\right)\sin\vartheta_x t\right.\\&\left.-\varphi_{f2}\vartheta_x\vartheta_y\left(\left(B_{nj}\cosh\vartheta_y t+C_{nj}\sinh\vartheta_y t\right)\cos\vartheta_x t-B_{nj}\cos\varphi_{f2}t\right)-C_{nj}\vartheta_x\left(\varphi_{f2}^2-\vartheta_y^2-\vartheta_x^2\right)\sin\varphi_{f2}t\right\}\end{aligned}$$

$$-\frac{2}{\varphi_{f1}\left[\left(\varphi_{f1}+\vartheta_x\right)^2+\vartheta_y^2\right]\left[\left(\varphi_{f1}-\vartheta_x\right)^2+\vartheta_y^2\right]}\left\{\varphi_{f1}\left(\varphi_{f1}^2-\omega_1\omega_2\right)\left(B_{nj}\sinh\vartheta_y t+C_{nj}\cosh\vartheta_y t\right)\sin\vartheta_x t\right.$$

$$\left.-\varphi_{f1}\vartheta_x\vartheta_y\left(\left(B_{nj}\cosh\vartheta_y t+C_{nj}\sinh\vartheta_y t\right)\cos\vartheta_x t-B_{nj}\cos\varphi_{f1}t\right)-C_{nj}\vartheta_x\left(\varphi_{f1}^2-\vartheta_y^2-\vartheta_x^2\right)\sin\varphi_{f1}t\right\}$$

$$+\frac{2}{\varphi_{f2}\left[\left(\varphi_{f2}+\vartheta_x\right)^2+\vartheta_y^2\right]\left[\left(\varphi_{f2}-\vartheta_x\right)^2+\vartheta_y^2\right]}\left\{\varphi_{f2}\left(\varphi_{f2}^2+\omega_1\omega_2\right)\left[\left(A_{ni}B_{nj}\sinh\vartheta_y t+A_{ni}C_{nj}\cosh\vartheta_y t\right)\right.\right.$$

$$\cos\vartheta_x t-A_{ni}C_{nj}\cos\varphi_{f2}t\Big]+\varphi_{f2}\vartheta_x\vartheta_y\left(A_{ni}B_{nj}\cosh\vartheta_y t+A_{ni}C_{nj}\sinh\vartheta_y t\right)\sin\vartheta_x t$$

$$\left.-A_{ni}B_{nj}\vartheta_y\left(\varphi_{f2}^2+\vartheta_y^2+\vartheta_x^2\right)\sin\varphi_{f2}t\right\}$$

$$-\frac{2}{\varphi_{f1}\left[\left(\varphi_{f1}+\vartheta_x\right)^2+\vartheta_y^2\right]\left[\left(\varphi_{f1}-\vartheta_x\right)^2+\vartheta_y^2\right]}\left\{\varphi_{f1}\left(\varphi_{f1}^2-\omega_1\omega_2\right)\left[\left(A_{ni}B_{nj}\sinh\vartheta_y t+A_{ni}C_{nj}\cosh\vartheta_y t\right)\cdot\right.\right.$$

$$\cos\vartheta_x t-A_{ni}C_{nj}\cos\varphi_{f1}t\Big]+\varphi_{f1}\vartheta_x\vartheta_y\left(A_{ni}B_{nj}\cosh\vartheta_y t+A_{ni}C_{nj}\sinh\vartheta_y t\right)\sin\vartheta_x t$$

$$\left.-A_{ni}B_{nj}\vartheta_y\left(\varphi_{f1}^2+\vartheta_y^2+\vartheta_x^2\right)\sin\varphi_{f1}t\right\}$$

$$+\frac{2}{\varphi_{f2}\left[\left(\varphi_{f2}+\vartheta_y\right)^2+\vartheta_x^2\right]\left[\left(\varphi_{f2}-\vartheta_y\right)^2+\vartheta_x^2\right]}\left\{\varphi_{f2}\left(\varphi_{f2}^2+\omega_1\omega_2\right)\left[\left(B_{ni}\sin\vartheta_y t+B_{ni}A_{nj}\cos\vartheta_y t\right)\sinh\vartheta_x t\right.\right.$$

$$\left.+\varphi_{f2}\vartheta_x\vartheta_y\left(B_{ni}\cos\vartheta_y t-B_{ni}A_{nj}\sin\vartheta_y t\right)\cosh\vartheta_x t-B_{ni}\cos\varphi_{f2}t\Big]-B_{ni}A_{nj}\vartheta_x\left(\varphi_{f2}^2+\vartheta_x^2+\vartheta_y^2\right)\sin\varphi_{f2}t\right\}$$

$$-\frac{2}{\varphi_{f1}\left[\left(\varphi_{f1}+\vartheta_y\right)^2+\vartheta_x^2\right]\left[\left(\varphi_{f1}-\vartheta_y\right)^2+\vartheta_x^2\right]}\left\{\varphi_{f1}\left(\varphi_{f1}^2+\omega_1\omega_2\right)\left[\left(B_{ni}\sin\vartheta_y t+B_{ni}A_{nj}\cos\vartheta_y t\right)\sinh\vartheta_x t\right.\right.$$

$$\left.+\varphi_{f1}\vartheta_x\vartheta_y\left(B_{ni}\cos\vartheta_y t-B_{ni}A_{nj}\sin\vartheta_y t\right)\cosh\vartheta_x t-B_{ni}\cos\varphi_{f1}t\Big]-B_{ni}A_{nj}\vartheta_x\left(\varphi_{f1}^2+\vartheta_x^2+\vartheta_y^2\right)\sin\varphi_{f1}t\right\}$$

$$+\left(B_{ni}B_{nj}+C_{ni}C_{nj}\right)\left[\frac{\left(\cosh\omega_1 t-\cos\varphi_{f2}t\right)}{\varphi_{f2}^2+\omega_1^2}-\frac{\left(\cosh\omega_2 t-\cos\varphi_{f1}t\right)}{\varphi_{f1}^2+\omega_2^2}\right]$$

$$-\left(B_{ni}B_{nj}-C_{ni}C_{nj}\right)\left[\frac{\left(\cosh\omega_2 t-\cos\varphi_{f2}t\right)}{\varphi_{f2}^2+\omega_2^2}-\frac{\left(\cosh\omega_1 t-\cos\varphi_{f1}t\right)}{\varphi_{f1}^2+\omega_1^2}\right]$$

$$+\frac{\left(B_{ni}C_{nj}+C_{ni}B_{nj}\right)}{\varphi_{f2}\left(\varphi_{f2}^2+\omega_2^2\right)}\left(\varphi_{f2}\sinh\omega_2 t-\omega_2\sin\varphi_{f2}t\right)+\frac{\left(B_{ni}C_{nj}-C_{ni}B_{nj}\right)}{\varphi_{f2}\left(\varphi_{f2}^2+\omega_1^2\right)}\left(\varphi_{f2}\sinh\omega_1 t-\omega_1\sin\varphi_{f2}t\right)$$

$$-\frac{\left(B_{ni}C_{nj}+C_{ni}B_{nj}\right)}{\varphi_{f1}\left(\varphi_{f1}^2+\omega_2^2\right)}\left(\varphi_{f1}\sinh\omega_2 t-\omega_2\sin\varphi_{f1}t\right)+\frac{\left(B_{ni}C_{nj}-C_{ni}B_{nj}\right)}{\varphi_{f1}\left(\varphi_{f1}^2+\omega_1^2\right)}\left(\varphi_{f1}\sinh\omega_1 t-\omega_1\sin\varphi_{f1}t\right)$$

$$+\frac{2}{\varphi_{f2}\left[\left(\varphi_{f2}+\vartheta_y\right)^2+\vartheta_x^2\right]\left[\left(\varphi_{f2}-\vartheta_y\right)^2+\vartheta_x^2\right]}\left\{\varphi_{f2}\left(\varphi_{f2}^2-\omega_1\omega_2\right)\left[\left(C_{ni}\sin\vartheta_y t+C_{ni}A_{nj}\cos\vartheta_y t\right)\right.\right.$$

$$\cosh\vartheta_x t-C_{ni}A_{nj}\cos\varphi_{f2}t\Big]-\varphi_{f2}\vartheta_x\vartheta_y\left(C_{ni}\cos\vartheta_y t-C_{ni}A_{nj}\sin\vartheta_y t\right)\sinh\vartheta_x t$$

$$-C_{ni}\vartheta_y\left(\varphi_{f2}^2-\vartheta_x^2-\vartheta_y^2\right)\sin\varphi_{f2}t\Big\}$$

$$-\frac{2}{\varphi_{f1}\left[\left(\varphi_{f1}+\vartheta_x\right)^2+\vartheta_y^2\right]\left[\left(\varphi_{f1}-\vartheta_x\right)^2+\vartheta_y^2\right]}\Big\{\varphi_{f1}\left(\varphi_{f1}^2-\omega_1\omega_2\right)\left(B_{nj}\sinh\vartheta_y t+C_{nj}\cosh\vartheta_y t\right)\sin\vartheta_x t$$

$$-\varphi_{f1}\vartheta_x\vartheta_y\left(\left(B_{nj}\cosh\vartheta_y t+C_{nj}\sinh\vartheta_y t\right)\cos\vartheta_x t-B_{nj}\cos\varphi_{f1}t\right)-C_{nj}\vartheta_x\left(\varphi_{f1}^2-\vartheta_y^2-\vartheta_x^2\right)\sin\varphi_{f1}t\Big\}$$

$$+\frac{2}{\varphi_{f2}\left[\left(\varphi_{f2}+\vartheta_x\right)^2+\vartheta_y^2\right]\left[\left(\varphi_{f2}-\vartheta_x\right)^2+\vartheta_y^2\right]}\Big\{\varphi_{f2}\left(\varphi_{f2}^2+\omega_1\omega_2\right)\Big[\left(A_{ni}B_{nj}\sinh\vartheta_y t+A_{ni}C_{nj}\cosh\vartheta_y t\right)\cdot$$

$$\cos\vartheta_x t-A_{ni}C_{nj}\cos\varphi_{f2}t\Big]+\varphi_{f2}\vartheta_x\vartheta_y\left(A_{ni}B_{nj}\cosh\vartheta_y t+A_{ni}C_{nj}\sinh\vartheta_y t\right)\sin\vartheta_x t$$

$$-A_{ni}B_{nj}\vartheta_y\left(\varphi_{f2}^2+\vartheta_y^2+\vartheta_x^2\right)\sin\varphi_{f2}t\Big\}$$

$$-\frac{2}{\varphi_{f1}\left[\left(\varphi_{f1}+\vartheta_x\right)^2+\vartheta_y^2\right]\left[\left(\varphi_{f1}-\vartheta_x\right)^2+\vartheta_y^2\right]}\Big\{\varphi_{f1}\left(\varphi_{f1}^2-\omega_1\omega_2\right)\Big[\left(A_{ni}B_{nj}\sinh\vartheta_y t+A_{ni}C_{nj}\cosh\vartheta_y t\right)$$

$$\left(\cos\vartheta_x t-A_{ni}C_{nj}\cos\varphi_{f1}t\right)\Big]+\varphi_{f1}\vartheta_x\vartheta_y\left(A_{ni}B_{nj}\cosh\vartheta_y t+A_{ni}C_{nj}\sinh\vartheta_y t\right)\sin\vartheta_x t$$

$$-A_{ni}B_{nj}\vartheta_y\left(\varphi_{f1}^2+\vartheta_y^2+\vartheta_x^2\right)\sin\varphi_{f1}t\Big\}$$

$$+\frac{2}{\varphi_{f2}\left[\left(\varphi_{f2}+\vartheta_y\right)^2+\vartheta_x^2\right]\left[\left(\varphi_{f2}-\vartheta_y\right)^2+\vartheta_x^2\right]}\Big\{\varphi_{f2}\left(\varphi_{f2}^2+\omega_1\omega_2\right)\Big[\left(B_{ni}\sin\vartheta_y t+B_{ni}A_{nj}\cos\vartheta_y t\right)\sinh\vartheta_x t$$

$$+\varphi_{f2}\vartheta_x\vartheta_y\left(B_{ni}\cos\vartheta_y t-B_{ni}A_{nj}\sin\vartheta_y t\right)\cosh\vartheta_x t-B_{ni}\cos\varphi_{f2}t\Big]-B_{ni}A_{nj}\vartheta_x\left(\varphi_{f2}^2+\vartheta_x^2+\vartheta_y^2\right)\sin\varphi_{f2}t\Big\}$$

(16)

### 1.2.3 Solution of the Transformed Equation for the Moving Mass Problem

$$U(x,y,t)=\sum_{n_i}^{\infty}\sum_{n_j}^{\infty}\frac{1}{\left(\varphi_{m1}^2-\varphi_{m2}^2\right)}\left(\frac{P_f}{2}\left\{\left[\frac{1+A_{ni}A_{nj}}{\varphi_{m2}^2-\omega_1^2}\left(\cos\omega_1 t-\cos\varphi_{m2}t\right)\right.\right.\right.$$

$$\left.-\frac{1-A_{ni}A_{nj}}{\varphi_{m2}^2-\omega_2^2}\left(\cos\omega_2 t-\cos\varphi_{m2}t\right)-\frac{1+A_{ni}A_{nj}}{\varphi_{m1}^2-\omega_1^2}\left(\cos\omega_1 t-\cos\varphi_{m1}t\right)-\frac{1-A_{ni}A_{nj}}{\varphi_{m1}^2-\omega_2^2}\left(\cos\omega_2 t-\cos\varphi_{m1}t\right)\right]$$

$$+\frac{1}{\varphi_{m2}}\left[\frac{A_{nj}-A_{ni}}{\varphi_{m2}^2-\omega_1^2}\left(\varphi_{m2}\sin\omega_1 t-\omega_1\sin\varphi_{m2}t\right)+\frac{A_{nj}+A_{ni}}{\varphi_{m2}^2-\omega_2^2}\left(\varphi_{m2}\sin\omega_2 t-\omega_2\sin\varphi_{m2}t\right)\right]$$

$$+\frac{1}{\varphi_{m1}}\left[\frac{A_{nj}-A_{ni}}{\varphi_{m1}^2-\omega_1^2}\left(\varphi_{m1}\sin\omega_1 t-\omega_1\sin\varphi_{m1}t\right)+\frac{A_{nj}+A_{ni}}{\varphi_{m1}^2-\omega_2^2}\left(\varphi_{m1}\sin\omega_2 t-\omega_2\sin\varphi_{m1}t\right)\right]$$

$$+\frac{2}{\varphi_{m2}\left[\left(\varphi_{m2}+\vartheta_x\right)^2+\vartheta_y^2\right]\left[\left(\varphi_{m2}-\vartheta_x\right)^2+\vartheta_y^2\right]}\Big\{\varphi_{m2}\left(\varphi_{m2}^2-\omega_1\omega_2\right)\left(B_{nj}\sinh\vartheta_y t+C_{nj}\cosh\vartheta_y t\right)\sin\vartheta_x t$$

$$-\varphi_{m2}\vartheta_x\vartheta_y\left(\left(B_{nj}\cosh\vartheta_y t+C_{nj}\sinh\vartheta_y t\right)\cos\vartheta_x t-B_{nj}\cos\varphi_{m2}t\right)-C_{nj}\vartheta_x\left(\varphi_{m2}^2-\vartheta_y^2-\vartheta_x^2\right)\sin\varphi_{m2}t\Big\}$$

$$-\frac{2}{\varphi_{m1}\left[\left(\varphi_{m1}+\vartheta_x\right)^2+\vartheta_y^2\right]\left[\left(\varphi_{m1}-\vartheta_x\right)^2+\vartheta_y^2\right]}\left\{\varphi_{m1}\left(\varphi_{m1}^2-\omega_1\omega_2\right)\left(B_{nj}\sinh\vartheta_y t+C_{nj}\cosh\vartheta_y t\right)\sin\vartheta_x t\right.$$

$$\left.-\varphi_{m1}\vartheta_x\vartheta_y\left(\left(B_{nj}\cosh\vartheta_y t+C_{nj}\sinh\vartheta_y t\right)\cos\vartheta_x t-B_{nj}\cos\varphi_{m1}t\right)-C_{nj}\vartheta_x\left(\varphi_{m1}^2-\vartheta_y^2-\vartheta_x^2\right)\sin\varphi_{m1}t\right\}$$

$$+\frac{2}{\varphi_{m2}\left[\left(\varphi_{m2}+\vartheta_y\right)^2+\vartheta_x^2\right]\left[\left(\varphi_{m2}-\vartheta_y\right)^2+\vartheta_x^2\right]}+\frac{2}{\varphi_{m2}\left[\left(\varphi_{m2}+\vartheta_x\right)^2+\vartheta_y^2\right]\left[\left(\varphi_{m2}-\vartheta_x\right)^2+\vartheta_y^2\right]}$$

$$\left\{\varphi_{m2}\left(\varphi_{m2}^2+\omega_1\omega_2\right)\left[\left(A_{ni}B_{nj}\sinh\vartheta_y t+A_{ni}C_{nj}\cosh\vartheta_y t\right)\right.\right.$$

$$\left.\cos\vartheta_x t-A_{ni}C_{nj}\cos\varphi_{m2}t\right]+\varphi_{m2}\vartheta_x\vartheta_y\left(A_{ni}B_{nj}\cosh\vartheta_y t+A_{ni}C_{nj}\sinh\vartheta_y t\right)\sin\vartheta_x t$$

$$\left.-A_{ni}B_{nj}\vartheta_y\left(\varphi_{m2}^2+\vartheta_y^2+\vartheta_x^2\right)\sin\varphi_{m2}t\right\}$$

$$-\frac{2}{\varphi_{m1}\left[\left(\varphi_{m1}+\vartheta_x\right)^2+\vartheta_y^2\right]\left[\left(\varphi_{m1}-\vartheta_x\right)^2+\vartheta_y^2\right]}\left\{\varphi_{m1}\left(\varphi_{m1}^2-\omega_1\omega_2\right)\left[\left(A_{ni}B_{nj}\sinh\vartheta_y t+A_{ni}C_{nj}\cosh\vartheta_y t\right).\right.\right.$$

$$\left(\cos\vartheta_x t-A_{ni}C_{nj}\cos\varphi_{m1}t\right]\Big)+\varphi_{m1}\vartheta_x\vartheta_y\left(A_{ni}B_{nj}\cosh\vartheta_y t+A_{ni}C_{nj}\sinh\vartheta_y t\right)\sin\vartheta_x t$$

$$\left.-A_{ni}B_{nj}\vartheta_y\left(\varphi_{m1}^2+\vartheta_y^2+\vartheta_x^2\right)\sin\varphi_{m1}t\right\}\left\{\varphi_{m2}\left(\varphi_{m2}^2+\omega_1\omega_2\right)\left[\left(B_{ni}\sin\vartheta_y t+B_{ni}A_{nj}\cos\vartheta_y t\right)\sinh\vartheta_x t\right.\right.$$

$$\left.+\varphi_{m2}\vartheta_x\vartheta_y\left(B_{ni}\cos\vartheta_y t-B_{ni}A_{nj}\sin\vartheta_y t\right)\cosh\vartheta_x t-B_{ni}\cos\varphi_{m2}t\right]-B_{ni}A_{nj}\vartheta_x\left(\varphi_{m2}^2+\vartheta_x^2+\vartheta_y^2\right)\sin\varphi_{m2}t\Big\}$$

$$-\frac{2}{\varphi_{m1}\left[\left(\varphi_{m1}+\vartheta_y\right)^2+\vartheta_x^2\right]\left[\left(\varphi_{m1}-\vartheta_y\right)^2+\vartheta_x^2\right]}\left\{\varphi_{m1}\left(\varphi_{m1}^2+\omega_1\omega_2\right)\left[\left(B_{ni}\sin\vartheta_y t+B_{ni}A_{nj}\cos\vartheta_y t\right)\sinh\vartheta_x t\right.\right.$$

$$\left.+\varphi_{m1}\vartheta_x\vartheta_y\left(B_{ni}\cos\vartheta_y t-B_{ni}A_{nj}\sin\vartheta_y t\right)\cosh\vartheta_x t-B_{ni}\cos\varphi_{m1}t\right]-B_{ni}A_{nj}\vartheta_x\left(\varphi_{m1}^2+\vartheta_x^2+\vartheta_y^2\right)\sin\varphi_{m1}t\Big\}$$

$$+\left(B_{ni}B_{nj}+C_{ni}C_{nj}\right)\left[\frac{\left(\cosh\omega_1 t-\cos\varphi_{m2}t\right)}{\varphi_{m2}^2+\omega_1^2}-\frac{\left(\cosh\omega_2 t-\cos\varphi_{m1}t\right)}{\varphi_{m1}^2+\omega_2^2}\right]$$

$$-\left(B_{ni}B_{nj}-C_{ni}C_{nj}\right)\left[\frac{\left(\cosh\omega_2 t-\cos\varphi_{m2}t\right)}{\varphi_{m2}^2+\omega_2^2}-\frac{\left(\cosh\omega_1 t-\cos\varphi_{m1}t\right)}{\varphi_{m1}^2+\omega_1^2}\right]$$

$$+\frac{\left(B_{ni}C_{nj}+C_{ni}B_{nj}\right)}{\varphi_{m2}\left(\varphi_{m2}^2+\omega_2^2\right)}\left(\varphi_{m2}\sinh\omega_2 t-\omega_2\sin\varphi_{m2}t\right)+\frac{\left(B_{ni}C_{nj}-C_{ni}B_{nj}\right)}{\varphi_{m2}\left(\varphi_{m2}^2+\omega_1^2\right)}\left(\varphi_{m2}\sinh\omega_1 t-\omega_1\sin\varphi_{m2}t\right)$$

$$-\frac{\left(B_{ni}C_{nj}+C_{ni}B_{nj}\right)}{\varphi_{m1}\left(\varphi_{m1}^2+\omega_2^2\right)}\left(\varphi_{m1}\sinh\omega_2 t-\omega_2\sin\varphi_{m1}t\right)+\frac{\left(B_{ni}C_{nj}-C_{ni}B_{nj}\right)}{\varphi_{m1}\left(\varphi_{m1}^2+\omega_1^2\right)}\left(\varphi_{m1}\sinh\omega_1 t-\omega_1\sin\varphi_{m1}t\right)$$

$$+\frac{2}{\varphi_{m2}\left[\left(\varphi_{m2}+\vartheta_y\right)^2+\vartheta_x^2\right]\left[\left(\varphi_{m2}-\vartheta_y\right)^2+\vartheta_x^2\right]}\left\{\varphi_{m2}\left(\varphi_{m2}^2-\omega_1\omega_2\right)\left[\left(C_{ni}\sin\vartheta_y t+C_{ni}A_{nj}\cos\vartheta_y t\right)\cdot\right.\right.$$

$$\left.\cosh\vartheta_x t-C_{ni}A_{nj}\cos\varphi_{m2}t\right]-\varphi_{m2}\vartheta_x\vartheta_y\left(C_{ni}\cos\vartheta_y t-C_{ni}A_{nj}\sin\vartheta_y t\ \sinh\vartheta_x t\right)$$

$$\left.-C_{ni}\vartheta_y\left(\varphi_{m2}^2-\vartheta_x^2-\vartheta_y^2\right)\sin\varphi_{m2}t\right\}$$

$$
-\frac{2}{\varphi_{m1}\left[\left(\varphi_{m1}+\vartheta_y\right)^2+\vartheta_x^2\right]\left[\left(\varphi_{m1}-\vartheta_y\right)^2+\vartheta_x^2\right]}\Big\{\varphi_{m1}\left(\varphi_{m1}^2+\omega_1\omega_2\right)\Big[\left(C_{ni}\sin\vartheta_y t+C_{ni}A_{nj}\cos\vartheta_y t\right)
$$

$$
\cosh\vartheta_x t-C_{ni}A_{nj}\cos\varphi_{m1}t\Big]-\varphi_{m1}\vartheta_x\vartheta_y\left(C_{ni}\cos\vartheta_y t-C_{ni}A_{nj}\sin\vartheta_y t\right)\sinh\vartheta_x t
$$

$$
-C_{ni}\vartheta_y\left(\varphi_{m1}^2-\vartheta_x^2-\vartheta_y^2\right)\sin\varphi_{m1}t\Big\}+\Big[\left(1-w_{m1}^2\right)\cos w_{m1}t-\left(1-w_{m2}\right)\cos w_{m2}t\Big]y_0\Big)
$$

$$
\cdot\left(\sin\psi_{ni}x+A_{ni}\cos\psi_{ni}x+B_{ni}\sinh\psi_{ni}x+C_{ni}\cosh\psi_{ni}x\right)
$$

$$
\cdot\left(\sin\psi_{nj}y+A_{nj}\cos\psi_{nj}y+B_{nj}\sinh\psi_{nj}y+C_{nj}\cosh\psi_{nj}y\right) \qquad (17)
$$

Equation (17) is the transverse displacement for the general boundary conditions for moving mass problem of the anisotropic plate on Vlasov foundation subject to a moving load described in equation (16).

### 1.2.4 Application

In order to illustrate the problem and results obtained practically. The following plate end conditions are the commonly used referred to as the classical end conditions namely; simply supported, clamped, cantilever and the simple-free end conditions.

### 1.2.5 Simple-Free-Simple- Free condition

The mathematical representation of the boundary condition for modeling a bridge is the *Simple- Free- Simple- Free condition* ends. In this illustrative example, the rectangular plate assumed to be simple on the edge $(u, u'') = 0$ at $x = 0, a$ and free on the edges $(u'', u''') = 0$ at $y = 0, b$ have free edge at $y = 0$, '$y = b$ .The boundary conditions are given as;

$$
\left.\begin{aligned}
&U(0,y,t)=U(a,y,t)=0;\quad U_{xx}(0,y,t)=U_{xx}(a,y,t)=0\\
&U_{yy}(x,0,t)=U_{yy}(x,b,t)=0;\quad U_{yyy}(x,0,t)=U_{yyy}(x,b,t)=0
\end{aligned}\right\}\quad (16)
$$


(i) (ii) (iii)

Figure 6 : (i) Simply supported (ii) Free (iii) Clamped

# Simple - Free - Simple - Free Graphs

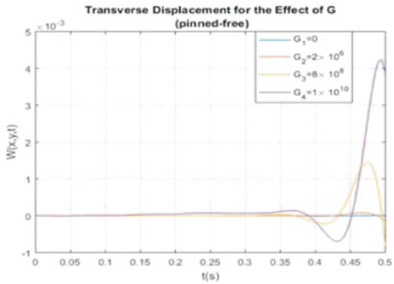



FIG 2A: Effect of shear modulus (G)

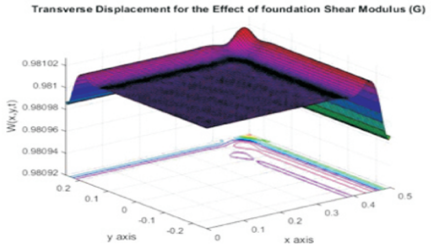



*FIG2B* : $G = 0$

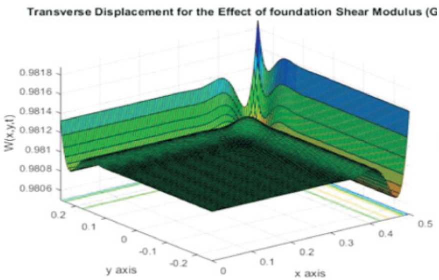



*FIG2C* : $G = 8 \times 10^4$

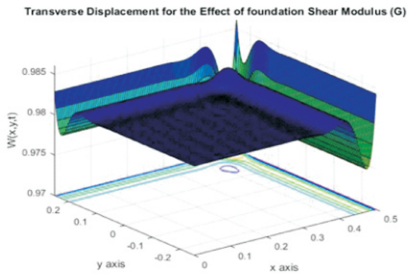
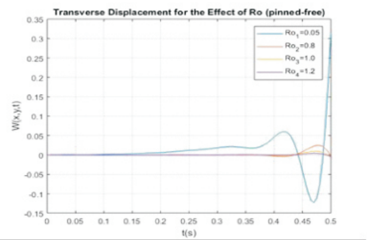



*FIG2D* : $G = 8 \times 10^6$



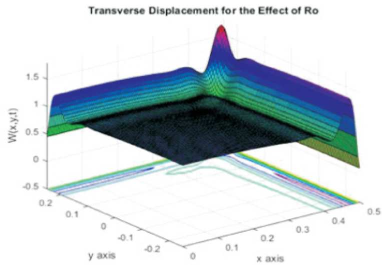




*FIG 2E:* $R_0 = 0.5$

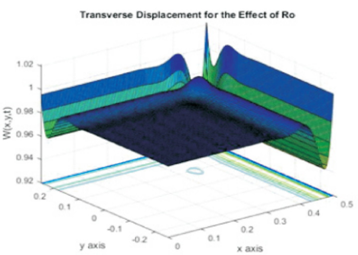



*FIG2F* : $R_0 = 0.8$

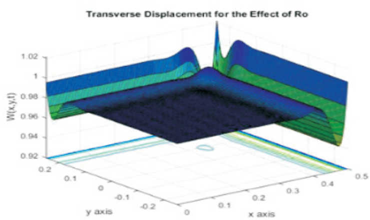



*FIG2F* : $R_0 = 1.2$

## 2. Application in Fluid Dynamics

### 2.1 Unsteady Magnetohydrodynamic (MHD) Thin Film Flow of a Third Grade Fluid

In recent years, considerable interest has been developed in the study of the flow of non- Newtonian fluid down an inclined plane because of its important applications in science, engineering and technology. Examples of their application can be found in ink-jet print, polymer processing, silvering processing, plasma enhanced chemical vapor deposition as well as in magneto-rheological thin film deposition.

Because of the complexity of non-Newtonian fluids, it is very difficult to analyze a single model that exhibits all its properties. The normal stress differences are described in second grade of non-Newtonian fluid, but it cannot predict shear thinning or thickening properties due to its constant apparent viscosity. The third grade fluid model attempt to include such characteristics of visco-elastics fluids.

#### 2.1.1 Heat transfer down an inclined plane

Following Aiyesimi *et al.* (2013) on MHD thin film flow of a third grade fluid down an inclined plane and retaining the parameters thereof our present initial boundary value thermodynamic problem is governed by the model;

$$\rho\frac{\partial u}{\partial t}=\mu\frac{\partial^2 u}{\partial y^2}+\alpha_1\frac{\partial^3 u}{\partial y^2\partial t}+6\left(\beta_2+\beta_3\right)\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2 u}{\partial y^2}-\sigma B_0^2 u+\rho g\sin\theta \qquad (2.1)$$

$$\rho c_p\frac{\partial T}{\partial t}=K\frac{\partial^2 T}{\partial y^2}+\mu\left(\frac{\partial u}{\partial y}\right)^2+\alpha_1\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y\partial t}+2\left(\beta_2+\beta_3\right)\left(\frac{\partial u}{\partial y}\right)^4+\sigma B_0^2 u^2 \qquad (2.2),$$

$$\left.\begin{aligned}&u(0,t)=u_y(\delta,t)=0,\ 0\prec\delta\prec 1,\ t\succ 0\\&T(0,t)=T_W,\ T(\delta,t)=T_\delta,\ T(y,0)=0,\ 0\prec y\prec\delta\end{aligned}\right\} \qquad (2.3)$$

where the parameters $\rho$, $B_0$, $\sigma$, $\alpha_1$, $\beta_2$, $\beta_3$, $c_p$ and $k$ all retain their usual meanings and $\theta$ indicates angle of inclination. $T$ is the variable temperature of the fluid, $T_w$ is the temperature of the plane and $T_\delta$ is the temperature of the ambient fluid.

We define some non-dimensional quantities and following the basic requirement for the application of solution by perturbation method through existence of small or large parameter in equations, therefore it is taken that $\varepsilon=\beta$ as a small parameter and expand $u(\eta, t)$ in the Poincare-type series of the form $u(\eta, t) = u_0(\eta, t) + \varepsilon u_1(\eta, t)$ and $T(\eta, t) = \phi(\eta) + \theta(\eta, t)$

After some rigorous mathematical computations, we obtain the following results:

$$u(\eta, t) = \frac{-k\cosh\left(\eta\sqrt{M}-\sqrt{M}\right)+k\cosh\left(\sqrt{M}\right)}{M\cosh\left(\sqrt{M}\right)} + \sum_{n=1}^{\infty} b_n \sin\left[\frac{(2n-1)\pi}{2}\eta\right]\ell^{-\frac{(2n-1)^2\pi^2 t}{4}} + \varepsilon\sum_{n=1}^{\infty} c_{1n}(t)\, Y_{1n}(\eta) \qquad (2.5)$$

where

$$\left.\begin{aligned}
&b_n = 2\int_0^1 -\frac{-K\cos\left(\eta\sqrt{M}-\sqrt{M}\right)+K\cosh\left(\sqrt{M}\right)}{M\cosh\left(\sqrt{M}\right)}\sin\frac{(2n-1)}{2}\pi\eta\, d\eta\\
&Y_{1n}(\eta) = \sin\left[\frac{(2n-1)\pi\eta}{2}\right], n=1, 2, \ldots \quad b_1 = \frac{4K\left(-4\ell^{2\sqrt{M}}M-4M\right)}{\pi M\left(\pi^2+4M+\ell^{2\sqrt{M}}\pi^2+4\ell^{2\sqrt{M}}M\right)}\\
&c_{11}(t) = \begin{pmatrix} \dfrac{a_1\ell^{t\left(\frac{1}{4}\pi^2+M\right)}}{\frac{\pi^2}{4}+M} - \dfrac{a_4\ell^{-t\left(\frac{1}{4}\pi^2+M\right)}}{\frac{\pi^2}{4}+M} - \dfrac{a_2\ell^{-t\left(\frac{1}{2}\pi^2+2M\right)}}{\frac{\pi^2}{2}+2M} + a_3 t\\ -\dfrac{a_1}{\frac{\pi^2}{4}+M} + \dfrac{a_4}{\frac{\pi^2}{4}+M} + \dfrac{a_2}{\frac{\pi^2}{2}+2M} \end{pmatrix}\ell^{-t\left(\frac{1}{4}\pi^2+M\right)}\\
&Y_{11}(\eta) = \sin\left[\frac{\pi\eta}{2}\right]
\end{aligned}\right\} \qquad (2.6)$$

$$
\left.
\begin{aligned}
&b_2 = \frac{4k\left(-4\ell^{2\sqrt{M}}M - 4M\right)}{\pi M\left(12M + 27\pi^2 + 27\ell^{2\sqrt{M}}\pi^2 + 12\ell^{2\sqrt{M}}M\right)} \\
&c_{12}(t) = \left[ a_8 t + \frac{a_5 \ell^{\frac{1}{4}\left(\pi^2+4M\right)}}{\frac{9}{4}\pi^2 + M} - \frac{a_6 \ell^{-\frac{1}{2}t\left(9\pi^2+4M\right)}}{\frac{9}{2}\pi^2 + 2M} - \frac{a_7 \ell^{-\frac{t}{4}\left(9\pi^2+4M\right)}}{\frac{9}{4}\pi^2 + M} \right. \\
&\left. - \frac{a_5}{\frac{9}{4}\pi^2 + M} + \frac{a_6}{\frac{9}{2}\pi^2 + 2M} + \frac{a_7}{\frac{9}{4}\pi^2 + M} \right] \ell^{-\frac{t}{4}\left(9\pi^2+4M\right)} \\
&Y_{12} = \sin\left(\frac{3}{2}\pi\eta\right)
\end{aligned}
\right\} (2.7)
$$

For computational purpose it is sufficient to truncate our approximations at $n$=2,hence;

$$
u(\eta,t) = \frac{-k\cosh\left(\eta\sqrt{M} - \sqrt{M}\right) + k\cosh\sqrt{M}}{M\cosh\sqrt{M}} + b_1 \sin\left(\frac{\pi\eta}{2}\right)\ell^{-\frac{\pi^2 t}{4} - Mt}
$$

$$
+ b_2 \sin\left(\frac{3\pi\eta}{2}\right)\ell^{-\frac{9\pi^2 t}{4} - Mt} + \sum_{n=3}^{\infty} b_n \sin\left[\frac{(2n-1)\pi}{2}\eta\right]\ell^{-\frac{(2n-1)^2\pi^2 t}{4}} + \varepsilon\left[c_{11}(t)\,Y_{11}(\eta) + c_{12}(t)\,Y_{12}(\eta)\right] \quad (2.8)
$$

Similarly,

$$
\theta(\eta,t) = \sum_{r=1}^{\infty} c_{rT}(t)\; Y_{rT}(\eta) \qquad (2.9)
$$

$$
\left.
\begin{aligned}
&c_{1T}(t) = \left[ \frac{a_9 \Pr \ell^{\frac{\pi^2 t}{\Pr}}}{\pi^2} - \frac{4a_{10}\Pr \ell^{-\frac{1}{4}t\frac{\left(4\pi^2+\pi^2\Pr+4M\Pr\right)}{\Pr}}}{-4\pi^2 + \pi^2\Pr + 4M\Pr} - \frac{2a_{11}\Pr \ell^{-\frac{1}{2}t\frac{\left(-2\pi^2+\pi^2\Pr+4M\Pr\right)}{\Pr}}}{-2\pi^2 + \pi^2\Pr + 4M\Pr} \right. \\
&\left. -\left(\frac{2}{\pi} + \frac{a_{10}\Pr}{\pi^2}\right) - \frac{4a_{10}\Pr}{-4\pi^2 + \pi^2\Pr + 4M\Pr} + \frac{2a_{11}\Pr}{-2\pi^2 + \pi^2\Pr + 4M\Pr} \right] \ell^{\frac{-\pi^2 t}{\Pr}} \\
&Y_{1T}(\eta) = \sin(\pi\eta) \\
&c_{2T}(t) = \left[ \frac{1}{4}\frac{a_{12}\Pr \ell^{\frac{4\pi^2 t}{\Pr}}}{\pi^2} - \frac{2a_{13}\Pr \ell^{-\frac{1}{2}t\frac{\left(-8\pi^2+\pi^2\Pr+4M\Pr\right)}{\Pr}}}{-8\pi^2 + \pi^2\Pr + 4M\Pr} - \frac{4a_{14}\Pr \ell^{-\frac{1}{4}t\frac{\left(-16\pi^2+\pi^2\Pr+4M\Pr\right)}{\Pr}}}{-16\pi^2 + \pi^2\Pr + 4M\Pr} \right.
\end{aligned}
\right\} (2.10)
$$

$$+\frac{1}{\pi}-\frac{1}{4}\frac{a_{12}\Pr}{\pi^2}+\frac{2a_{13}\Pr}{-8\pi^2+\pi^2\Pr+4M\Pr}+\frac{4a_{14}\Pr}{-16\pi^2+\pi^2\Pr+4M\Pr}\Bigg]\ell^{\frac{-4\pi^2 t}{\Pr}}$$

$$Y_{2T}(\eta)=\sin(2\pi\eta)$$

$$\text{ie,}\,\theta(\eta,t)=\Bigg[\frac{a_9\Pr\ell^{\frac{\pi^2 t}{\Pr}}}{\pi^2}-\frac{4a_{10}\Pr\,\ell^{-\frac{\frac{1}{4}t\left(-4\pi^2+\pi^2\Pr+4M\Pr\right)}{\Pr}}}{-4\pi^2+\pi^2\Pr+4M\Pr}-\frac{2a_{11}\Pr\ell^{-\frac{\frac{1}{2}t\left(-2\pi^2+\pi^2\Pr+4M\Pr\right)}{\Pr}}}{-2\pi^2+\pi^2\Pr+4M\Pr}$$

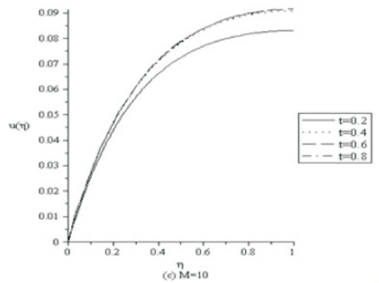
$$-\left(\frac{2}{\pi}+\frac{a_{10}\Pr}{\pi^2}\right)+\frac{4a_{10}\Pr}{-4\pi^2+\pi^2\Pr+4M\Pr}+\frac{2a_{11}\Pr}{-2\pi^2+\pi^2\Pr+4M\Pr}\Bigg]\ell^{\frac{-\pi^2 t}{\Pr}}\sin(\pi\eta)$$

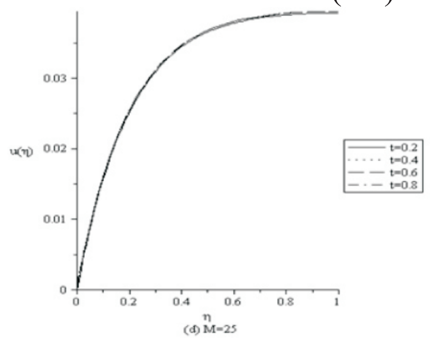
$$+\Bigg[\Bigg[\frac{1}{4}\frac{a_{12}\Pr\,\ell^{\frac{4\pi^2 t}{\Pr}}}{\pi^2}-\frac{2a_{13}\Pr\,\ell^{-\frac{\frac{1}{2}t\left(-8\pi^2+\pi^2\Pr+4M\Pr\right)}{\Pr}}}{-8\pi^2+\pi^2\Pr+4M\Pr}-\frac{4a_{14}\Pr\,\ell^{-\frac{\frac{1}{4}t\left(-16\pi^2+\pi^2\Pr+4M\Pr\right)}{\Pr}}}{-16\pi^2+\pi^2\Pr+4M\Pr}$$

$$+\frac{1}{\pi}-\frac{1}{4}\frac{a_{12}\Pr}{\pi^2}+\frac{2a_{13}\Pr}{-8\pi^2+\pi^2\Pr+4M\Pr}$$

$$+\frac{4a_{14}\Pr}{-16\pi^2+\pi^2\Pr+4M\Pr}\Bigg]\ell^{\frac{-4\pi^2 t}{\Pr}}+\sum_{r=3}^{\infty}\varepsilon^{r-1}c_{rT}(t)\,Y_{rT}(\eta)\Bigg]\sin(2\pi\eta)$$

$$T(\eta,t)=\eta+c_{1T}(t)\,Y_{1T}(\eta)+\varepsilon c_{2T} \qquad (2.11)$$




$\lambda = 0.1$

**Figure 7:** Time development of velocity profile $u$ for various values of $M$ and time $t$ when $K = 1$

### 2.1.2 An analytic investigation of convective boundary-layer flow of a Nanofluid past a stretching Sheet with radiation

According Aiyesimi *et al.* (2015), the steady two-dimensional boundary layer flow of a Nano-fluid past a stretching sheet in the presence of thermal effect and Radiation with the linear velocity $u = ax$, where $a$ , is constant, $x$ is the coordinate measured from the stretching sheet is zero is governed by

$$\nabla V = 0 \tag{2.12}$$

$$(V.\nabla)V = -\frac{1}{\rho}\nabla p + \nu\nabla^2 V \tag{2.13}$$

$$(V.\nabla)T = \alpha\nabla^2 T + \tau\left(D_B(\nabla C)\cdot(\nabla T) + \frac{D_T}{T_\infty}\left((\nabla T)^2 - 2T_xT_y\right)\right) - \frac{1}{\rho C_p} q_{r,y} \tag{2.14}$$

$$(V.\nabla)C = D_B\nabla^2 C + \left(\frac{D_T}{T_\infty}\right)\nabla^2 T - \sigma(C - C_\infty) \tag{2.15}$$

where

$$\left.\begin{array}{l} V = i\hat{u} + j\hat{v} \\ y = 0:\ u = ax, v = 0,\ T = T_W,\ C = C_W \\ y \to \infty: u = 0, v = 0,\ T \to T_\infty,\ C \to C_\infty \end{array}\right\} \tag{2.16}$$

where all the parameters retain their usual meanings. Invoking the optically thin formulation of Boussenesq, we have

$$T^4 \cong 4TT_\infty^3 - 3T_\infty^4 \tag{2.17}$$

$$\frac{\partial q_r}{\partial y} = -\frac{16T_\infty^{3*}\sigma}{3\delta}\frac{\partial^2 T}{\partial y^2} \tag{2.18}$$

**Using some similarity transformations, the problems are then reduced to non-linear coupled ordinary differential equations below:-**

$$f''' + ff'' - f'^2 + Gr_{Tx} + Gr_{Cx} = 0 \qquad (2.19)$$

$$\left(1 + \frac{4Ra}{3}\right)\theta'' + \Pr f\theta' + \Pr N_b\phi'\theta' + \Pr N_t\theta'^2 = 0 \qquad (2.20)$$

$$\phi'' + L_e f\phi' + \frac{N_t}{N_b}\theta'' - KS_C\phi = 0 \qquad (2.21)$$

with the corresponding conditions;

$$\left.\begin{array}{l} f(0)=0,\ f'(0)=1,\ \theta(0)=1, \phi(0)=0. \\ f'(\infty)=0, \theta(\infty)=0, \phi(\infty)=0. \end{array}\right\} \qquad (2.22)$$

where,

$$\left.\begin{array}{l} Gr_T = \dfrac{g\beta_0(T-T_\infty)}{a^2},\ Gr_C = \dfrac{g\beta_0(C-C_\infty)}{a^2} \\ Gr_{Tx} = \dfrac{g\beta(T-T_\infty)}{a^2x},\ Gr_{Cx} = \dfrac{g\beta(C-C_\infty)}{a^2x},\ Ra = \dfrac{4\sigma^* T_\infty^3}{\delta k^*} \\ P_r = \dfrac{\upsilon}{\alpha},\ L_e = \dfrac{\upsilon}{D_B},\ N_b = \dfrac{(\rho c)_p D_B(C_W - C_\infty)}{(\rho c)_f \upsilon}, \\ N_t = \dfrac{(\rho c)_p D_T(T_W - T_\infty)}{(\rho c)_f T_\infty \upsilon},\ S_c = \dfrac{\upsilon}{D_B},\ K = \dfrac{\sigma}{a} \end{array}\right\} (2.23)$$

the solution of (2.19) to (2.23) yield the similarity solutions.

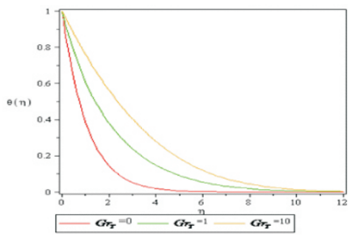



Fig 8a: Effect of $Gr_T$ on Temperature Profile

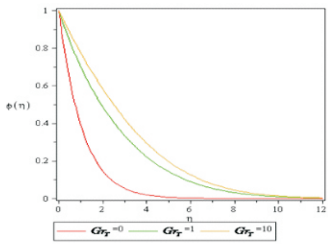



Fig 8b: Effect of $Gr_T$ on Concentration Profile

## Recommendation

According to Thomas Jefferson, "*An enlightened citizenry is indispensable for the proper functioning of a republic. Self – government is not possible unless the citizens are educated sufficiently to enable them to exercise oversight. It is therefore imperative that the nation see to it that a suitable education be provided for all its citizens*."

The term knowledge-based economy that has become a most used terminology in defining world economy is a modern day direct description of the Jefferson's effort in emphasizing the role of education in nation building.

Unfortunately, in our dear country Nigeria for decades now the education sector has gone comatose as it has completely been driven underground by paucity of funds. It is a public knowledge that even when smaller African nations are faithfully implementing the UNESCO recommendation of 26% budgetary allocation to the education sector Nigeria for over a decade now has consistently been unable to follow suit.. In fact, it is on record that the highest Nigeria has gone is in the neighborhoods of 10% budgetary allocation and in many instances as ridiculously low as 7% and unfortunately the situation even appears to be worsening over the years. This drift must be arrested for the sake of incoming generations.

There should also be a deliberate government policy to establish a synergy between the university system and the industry for the purpose of harnessing results from researches the institution for possible industrial revolution.

**Appreciation** To the King of Kings, Lord of Lords, the ancient of days, Eternal Rock of Ages, the great I am that 1am. The Conquering lion of the tribe of Judah. He has been so gracious, so merciful, loving and faithful to me. I owe him everything, He has been so loving and faithful to me and all that he has given me. He has made today to be.

To my parents: my late father Abubakar Jagun Aiyesimi who though never had the privilege of attending any formal school loved education so much, he was my main life motivator that at the very early part of my life when all hopes seemed lost for me to continue my education kept encouraging me in fact, he was the only person that have seen this day. Also to my mother who stood for me asa TRUE mother will for her child. I am so grateful to them.

I will also acknowledge the roles of my elder brothers: Late Chief D. I. Aiyesimi, Chief A. Berida and Mr. Oloniyo Aiyesimi played at various stages in my life. They had each played very undeniably noticeable and unique roles at different stages in my education pilgrimage.

To the family of Yomi Aiyesimi who is ably represented here by my amiable & loving wife Mrs. Grace Tola Aiyesimi. She has played perfectly the role of the mother of the home. Has taken very good care of thefamily as she has been graciously nursing the home to its present state as a professional nurse. I am equally indebted in gratitude to the lovely children, God has endowed us with: Emmanuel, Joseph, Godwin, Esther and Elizabeth. I love you all beyond ordinary expression.

I also want to use the opportunity to remember my late friend, Dr. Peter Shola Akinyeye. Up until hispremature death, he was a FRIEND indeed. Once more, I say may his gentle soul rest in peace PERFECTLY IN THE BOSOM OF God his creator (Amen).
I want to acknowledge my teachers from kindergarten to the university. I will not fail to remember my teachers from the beginning to the tertiary level, prominent among whom are: Baba Ali of blessed memory; though he might have been dead for over forty years now but that name started it all, late Professors. E.A Bagudu and Ibiejugba, Dr. Ogunsilure Professors P. Onumanyi, M.S. Audu and J. A Gbadeyan. Special mention must be made here of Prof. J.A Gbadeyan who was not just my lecturer in the university but my supervisor for both MSc and PhD, at the University of Ilorin. He played very prominent role in introducing me to the world of research and hence the main instrument to this present occasion, indeed he is THE ACADEMIC MENTOR OF PROFESSORS, I am so grateful and indebted to him. I also have to express my appreciation to my Almamater: The University of Ilorin that citadel of knowledge; I owe it my gratitude.

To all my colleagues in the Department of Mathematics of this institution I owe a lot of gratitude for the very fruitful professional and nonprofessional interactions that most time brought about challenging developments.

I want to appreciate ALL my students especially my research students with whom I have spent quality time. I am quite proud of them as they give me the needed professional fulfillment as a teacher.

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# Profile of the Inaugural Lecturer

Prof. Yomi Aiyesimi was born on the 10th of August 1959 to the humble family of late Abubakar Jagun Aiyesimi in Agbaja, in the present Lokoja Local Government Area of Kogi State.

He started his educational career at the Native Authority Primary School, Agbaja in 1964. After his primary education he had to remain at home for the next three years until he was admitted to Ogoro/ Morongo Teacher' s College, Ogoro in present Kogi State in 1974. After his Teacher Training programmer where he attained the Teacher" s Grade II Certificate in 1979, he taught briefly at St. Andrews II Primary School in Okene, Kogi State for a period of one year.

In 1981, he was admitted to the University of Ilorin, Ilorin Kara State for a Bachelor of Science Degree Programme in Mathematics. In 1985, he completed his Bachelor of Science (B.Sc) degree Programme and came out as the Best Graduating Student in Mathematics. He thereafter proceeded on the mandatory one-year National Youth Service Corp (NYSC) Programme at Government Science Secondary School Misau, Bauchi State as a Mathematics Teacher. On his return from NYSC assignment in 1986 he took up a teaching appointment with the then Kwara State Education Management Board (KWEMB) as Education Officer II and was posted to St. Augustine's College, Kabba. He served as a Mathematics Teacher. His quest for further studies propelled him to enroll for his Master of .Science programme (M.Sc) in Mathematics (Applied) at University of Ilorin in 1987. On completing his Master's degree programme he transferred his services to the Federal University of Technology Minna and assumed duty on the 9th of January, 1991 as an Assistant Lecturer.

At the Federal University of Technology Minna by dint of work and services he rose through the ranks to become a Professor of Mathematics on 1st October, 2009. He was appointed the Head of Department of Mathematics in October 2011, a position he held successfully for four years, a Department he played a very vital role in nurturing with other renowned academicians to its present enviable status in the University. He has also served the University in various committees and many assignments, which include but not limited to;

- University Postgraduate School Board (**2007-2008, 2012-2017**),
- Member of Senate Estimate and Budget Committee (April **2011- 2018**).
- Senate Representative on the Board of Postgraduate School (April **2011 to date**).
- Senate Representative on the Council Selection Board for the Appointment of a new Registrar (March **2012**).
- Member, Federal University of Technology, Minna Delegate to some universities in the United States of America (Dec. **2007**).
- Deputy Dean, School of Science and Science Education, Federal University of Technology, Minna (Aug. **2007-**NOV. **2008**).
- Chairman, Students' Union Election Committee, Federal University of Technology, Minna (**2008**)
- Member of University Senate (SSSE Congregation Representative), Federal University of Technology, Minna (**2004-2007**) etc.

Prof. Aiyesimi has had a very rewarding stay in FUT, Minna in respect of the number and caliber of research students he mentored to certification. As of this day he has produced a total of Fourteen(**14**) PhD graduates and many more Master of Technology graduates. Three of the PhD graduates have actually attained the rank of Professors of Mathematics. Unfortunately, we lost my first product in the person of Prof. Gideon Toyin Okedayo in late 2019. The Mathematics community lost a gem and a highly illustrious member. May his gentle soul rest in peace. Among the rest are senior academics in various polytechnics and universities most of whom I can joyfully say I introduced to the SCIENTIFIC RESEARCH world as I also supervised them at the M. Tech level. We recently established our research cluster group called THE APPLIED MATHEMATICS RESEARCH GROUP.