FEDERAL UNIVERSITY OF TECHNOLOGY
MINNA

PERISCOPING EPIDEMIOLOGY AND ECOLOGY WITH MATHEMATICAL TOOLS

By

NINUOLA IFEOLUWA AKINWANDE
B.Sc. (First Class Hons.), M.Sc. PhD (Ibadan)
Professor of Mathematics

INAUGURAL LECTURE SERIES 28

12TH JUNE, 2014

FEDERAL UNIVERSITY OF TECHNOLOGY
MINNA

PERISCOPING EPIDEMIOLOGY AND ECOLOGY WITH MATHEMATICAL TOOLS

By

NINUOLA IFEOLUWA AKINWANDE
B.Sc. (First Class Hons.), M.Sc. PhD (Ibadan)
Professor of Mathematics

INAUGURAL LECTURE SERIES 28

12TH JUNE, 2014

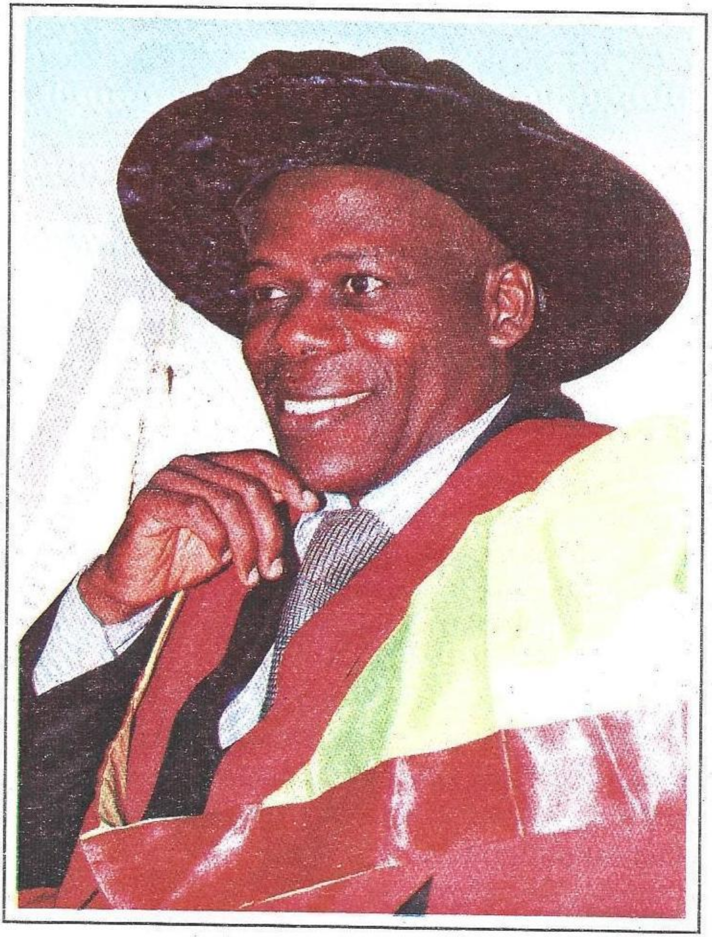
This 28th Inaugural Lecture was delivered under the Chairmanship of:

Professor M. A. Akanji, FNSMBM
Vice-Chancellor
Federal University of Technology, Minna

Published by:
University Seminar and Colloquium Committee
Federal University of Technology, Minna

12th June, 2014

Design + Print:
Global Links Communications - Nigeria
08056074844, 07036446818



NINUOLA IFEOLUWA AKINWANDE

B.Sc. (First Class Hons.), M.Sc. PhD (Ibadan)

Professor of Mathematics

SECTION ONE

1.0 INTRODUCTION

From the time of creation, populations of humans and animals communities or species, at various times, steadily increased or decreased and in some cases go into extinction due to several factors such as civilization, war, epidemics, migration, predation, harvesting and other ecological factors; cities and settlements have risen, spread or diminished over the years. For example, there were some settlements which were wiped out at the phenomenal outbreak of HIV/AIDS in some parts of Africa; and others through wars and famine or a combination of them. Other sparsely populated settlements have also transformed into densely populated community due to economic migration and political expediencies, a case in view is the Federal Capital Territory, Abuja, Nigeria. Prior to 1976, the area was a cluster of peasants' settlements while today it is densely populated.

Mathematicians have made various efforts to represent the growth or decay of populations with Mathematical formulae, using the Differential or Difference Equations, depending on whether the Population growth dynamics is treated as continuous or discrete. In Deterministic Mathematical Modelling, Ordinary and Partial Differential Equations are utilized to model Population Dynamics. In this wise, various living organisms which have the ability to reproduce, increase or in some cases go into total extinction can be objects of Mathematical Modelling; this field of study is currently referred to as Bio-Mathematics or Mathematical Biology. The off-shoots from this are Epidemiological and Health Sciences

Modelling or broadly speaking, Mathematical Ecology. Our research work include:

(i) Prey-Predator Interaction Models.
(ii) Water surface plants or animals like the Algae.
(iii) Bird flu and Animal Populations models.
(iv) Pollutant and oxygenation levels in a flowing river.
(v) Competition.
(vi) Cooperation.
(vii) Dynamics of Diseases – HIV, Malaria, Yellow Fever, SERS, Ebola Fever, Lassa Fever, Hepatitis B virus, etc.

Such Mathematical Models are then solved analytically where mathematically feasible, as demonstrated in some of our works. In several cases tools of Advanced Calculus and Functional Analysis are employed to qualitatively study the dynamics. Numerical or Computer Simulations of such models can be carried out, all these aimed at giving insight into the dynamics of the population concerned.

1.1 Overview of Mathematical Modelling

The term model is used in many different situations and in many different ways. Model may be defined as a simplified or idealized descriptions or conception of a particular system, situation or process. They may be categorized according to the medium in which they are expressed. Some types of models that have been identified in the philosophy of science literature are:

(a) Physical models including scale models and prototypes;
(b) Mathematical and Computational models.

Benyah (2009) defined Mathematical Modelling as "the process of creating a mathematical representation of some phenomenon in order to gain a better understanding of that phenomenon." Mathematical Modelling has become an important scientific technique over the last two decades and is becoming more and more a powerful tool to solve problems arising from science, engineering, industries and the society in general.

Characteristics of any modeling process are to identify important parameters which feature prominently in the dynamics. It allows us to generate new hypothesis, suggest experiments, and measure crucial parameters. Essentially, any real situation in the physical and biological world, whether natural or involving technology and human intervention is subject to analysis by modelling if it can be described in quantitative terms.

Once a model has been developed and applied to the problem, the resulting solution will then be analyzed and interpreted with respect to the problem. The model interpretations and conclusion are often modified to obtain a more accurate reflection of the observed reality of that phenomenon. Mathematical modeling is an evolving process, as new insight is gained the process begins again as additional factors are considered. The success of a model depends on how easily it can be used and how accurate are its predictions. It is worthy of note however that the closer a Mathematical Model assumptions are to the reality of the dynamics, the more intractable and difficult the Mathematical Analysis, hence the need to simplify assumptions reasonably without loosing track of the dynamics of the problem at hand.

1.2 The Stages of Mathematical Modeling

Developing a mathematical model for a real-life situation requires a reasonable level of understanding of the underlying principles of the system to be modelled. During the process of building a model, the modeller will decide what factors are relevant to the problem and what factors can be de-emphasized. Different real-life problems may require very different methods of approach. A general approach to the formulation of a real-life problem in mathematical terms is outlined as follow:

(i) Identify the problem;
(ii) Identify the important variables and parameters;
(iii) Determine how the variables relate together; stating the assumptions;
(iv) Develop the equation(s) which express the relationship between the variables and constants;
(v) Analyze or solve the resulting mathematical problem;
(vi) Interpret the result relating to real-life;
(vii) Relax or stiffen assumptions and go through (iv) to (vi).

1.3 The Exponential Growth Model

If y is a dependent variable, x is an independent variable and the rate of growth or decay of y depends directly on its present state or value, then we write the ODE

$$\frac{dy}{dx} = \alpha\, y;\;\; y(x_0) = y_0; \qquad \alpha = \text{ constant} \qquad (1.1)$$

as the functional relation between the derivative of y and the variables x and y.

Equation (1.1) is called the exponential growth model and has the solution

$$y = y_0 \, e^{\alpha(x - x_0)} \tag{1.2}$$

If $\alpha > 0$ then y is steadily growing while if $\alpha < 0$ it is decaying or decreasing.

Graphically, these scenarios are presented in Figures 1.1 and 1.2 respectively.

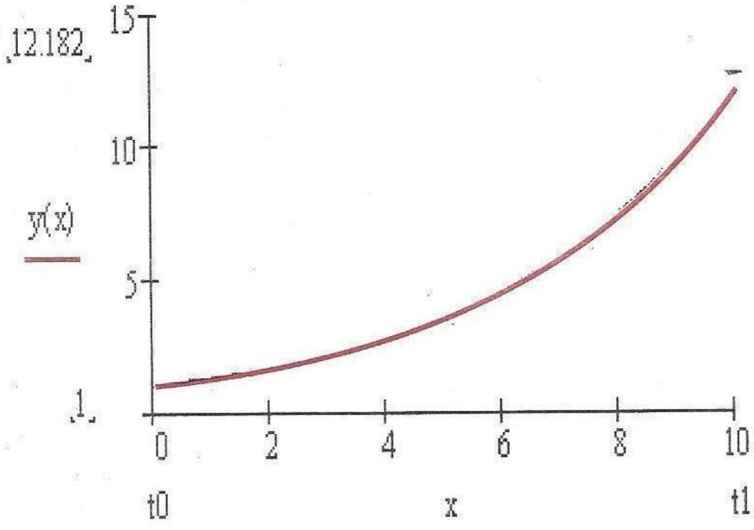


Figure 1.1 – Exponential growth model ($\alpha > 0$)

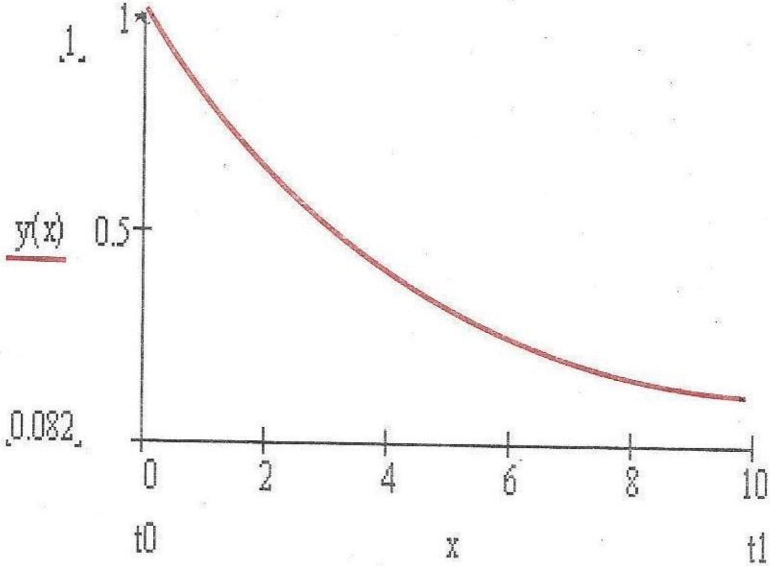


Figure 1.2 – Exponential decay model ($\alpha < 0$)

If $P(t)$ is the population of a community at time t, the Malthusian law which proposed the exponential growth model for populations is of the form in the equation (1.1) above given by:

$$\frac{dP(t)}{dt} = \delta\, P(t) \qquad (1.3)$$

where δ is the growth modulus. We note the following on the proposed exponential model equation (1.3):

If we define

β = birth rate

μ = death rate

then $\delta = \beta - \mu$

and the population $P(t)$ will grow or decay exponentially when $\delta > 0$ or $\delta < 0$ respectively.

1.4 The Logistic Population Model

The Logistic population model assumes that the environment has a carrying capacity denoted K, this is the maximum population which the environment can sustain; the model equation is given by Beltrami (1989):

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right); \qquad P(0) = P_0 \tag{1.4}$$

Where r is he per capita growth rate.

The analytical solution of (1.4) is given as:

$$P(t) = \frac{K P_0}{\left(K - P_0\right)e^{-rt} + P_0} \tag{1.5}$$

(i) This model has been successfully applied to several population dynamics with a great measure of success.

(ii) The model equation has equilibrium at the points $P(0) = 0$ and $P(t) = K$; note that the equilibrium or steady state of the model is when the rate of change is zero, i.e. when

$$\frac{dP}{dt} = 0 \tag{1.6}$$

(iii) The equilibrium state $P(0)=0$ is unstable, while the state $P(t)=K$ is stable.

(iv) A graphical representation of the logistic model is given below in figure 3.

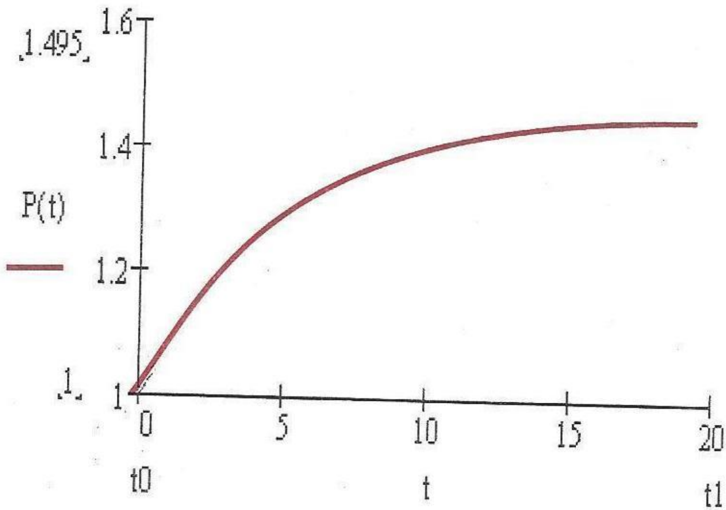


Figure 1.3 – The logistic population model; for P(0) < K

1.5 Age-Structured Mathematical Modelling

Modeling of physical situations in which consideration is given to time and space leads to Partial Differential Equations rather than Ordinary Differential Equation. In the case of population models, the space can be age grade. It can still be space, i.e. land mass, but in this work consideration is given only to age.

If $q(t, a)$ is the population of those in the age bracket or age grade $[a, a+da]$ at time t, then the total population $P(t)$ at time t is given by the integral value:

$$P(t) = \int_0^{\infty} q(t, a)\, da \tag{1.7}$$

An interesting aspect of this model is that the age and time both increase with the same value over the time interval. If we consider members of the population with age a at time t, if the time t increased by h units, then the group will increase in age by the same quantity h units, hence the differential is given by:

$$\frac{\partial q(t, a)}{\partial t} + \frac{\partial q(t, a)}{\partial a} = \lim_{h \to 0} \frac{q(t+h, a+h) - q(t, a)}{h} \tag{1.8}$$

From the principle of conservation, the rate of change given by (1.8) add to the rate d of individuals (per unit age and time) of age a who die at time t must equal zero.

When d is independent of the total population, it only depends on the time and age, and is given by:

$$d = \lambda(a) q(t, a) \tag{1.9}$$

while in the model by Gurtin & Mac Camy (1974) d depends on the total population and is given by:

$$d = \lambda(P(t), a) q(t, a) \tag{1.10}$$

The model equations are given by the set of equations (1.11) – (1.14) next

$$\frac{\partial q(t,a)}{\partial t}+\frac{\partial q(t,a)}{\partial a}+\lambda(P(t),a)q(t,a)=0 \qquad (1.11)$$

$$P(t)=\int_0^{\infty} q(t,a)\,da \qquad (1.12)$$

$$q(t,0)=\int_0^{\infty} \beta(P(t),a)q(t,a)\,da \qquad (1.13)$$

$$q(0,a)=\phi(a) \qquad (1.14)$$

referred to as a boundary value problem in Partial Differential Equations

1. The model equations presented in equations (1.11) – (1.14) above are referred to as the age-structured population model.
2. The terms $\beta(P(t),a)$ and $\lambda(P(t),a)$ are the birth and death modulus respectively.

SECTION TWO

THE NON-LINEAR AGE-STRUCTURED POPULATION MODEL

2.1 Introduction

In this section, we present our work on the Non-Linear Age Structured Population Model.

2.2 Non-Linear Age-Structured Population Model

Mac Camy & Gurtin (1974) proposed the Age-Structured Population Model. The authors then discussed existence of solution and obtained conditions for the stability of the equilibrium state. We have taken time to study this model, in our work, We

(i) Introduced finite life span $\Omega < \infty$, since all living organisms have finite life span and in line with the scriptures, "it is appointed onto man to die once after which there is judgement" Hebrews 9:27.

(ii) Solved the model equations thereby obtaining the Analytical Solution giving the population profile with respect to time. With this the population of a community of living organism could be projected over a period of time.

(iii) Derived/proposed two other methods with which we analyzed the equilibrium states for stability, we thus obtained stronger/better conditions in the form of constraint inequalities on the vital parameters of the model for the stability. We present a summary of these exploits in this section.

2.3 The Analytical Solution

We obtained the analytical solution of the one-dimensional non-linear age-structured population model equations (1.11) to (1.14). We utilized the theory of convolution, Apostol (1964), Akinwande (2004), Akinwande (2005), to solve the simultaneous integral equations arising which involve the total population P(t) and the total birth B(t) at time t which are given by:

$$P(t) = \int_0^t B(a)\exp\left\{-\int_0^{t-a}\mu(s)ds\right\}da$$

$$+\int_0^\infty \phi(a)\exp\left\{-\int_0^t \mu(a+s)ds\right\}da \tag{2.1}$$

and

$$B(t) = \int_0^\infty \beta(a-t)\phi(a)\exp\left\{-\int_0^t \mu(a+s)ds\right\}da$$

$$-\int_0^t\left[\beta(t-u)\exp\left\{-\int_0^{t-u}\mu(s)ds\right\}\right.$$

$$\left.\int_0^\infty \beta(a-u)\phi(a)\exp\left\{-\int_0^u \mu(a+s)ds\right\}da\right]du \tag{2.2}$$

respectively.

We note here that Gurtin and Mac Camy (1974) proved existence of solution while we obtained the solution. This can enable us project the total population and birth of a community of organisms over time where the parameters of birth and death rates are available and reliable.

2.4 Stability Analysis (First Method)

We applied the theorem by Bellman & Cooke (1963) and our analysis gave the inequalities

$$\beta_0' \leq 0. \qquad \mu_0' \leq 0. \qquad (2.3)$$

From which we deduced as the necessary conditions for the stability of the equilibrium state of the model.

For this same example, the result obtained by Gurtin and Mac Camy (1974) is $\mu_0' > \beta_0'$ as a condition for stability. Thus we conclude that the inequality $\beta_0' < \mu_0' < 0$ gives better necessary condition for the stability of equilibrium state which is an enhanced constraint.

2.5 Stability Analysis (Second Method - COA-Sowunmi's Lemma)

In the search for improved method of obtaining conditions for stability, we proposed, proved and applied the COA-Sowunmi's Lemma. This result was prepared into a small booklet in honour of my mentor and academic supervisor Late Professor Charles Olusegun Adedayo Sowunmi (1934 - 2007), Professor of Mathematics, University of Ibadan.

The book was presented at the occasion of the launch of his biography at the University of Ibadan on December 3, 2013. A summary of the work is presented below.

Lemma (COA-Sowunmi's Lemma)

Suppose the characteristic equation arising from the perturbation of the equilibrium state of a dynamical system is of the transcendental form

$$\int_0^{\infty} e^{-\lambda t} W(t) dt = 1 \tag{2.4}$$

where λ is the eigenvalue and $W(t)$ is a continuous function of t. Let

$$g(u) = \int_0^{\infty} e^{-ut} |W(t)| dt \tag{2.5}$$

be the real part of the expression obtained from the left hand side of (2.4) by setting $\lambda = u \pm iv$, then a necessary condition for the local asymptotic stability of the equilibrium state of the system is given by

$$g(0) < 1 \tag{2.6}$$

The equilibrium or steady state is unstable if

$$g(0) > 1 \tag{2.7}$$

We applied the Lemma to study the stability of the equilibrium state of the Non-Linear Age-Structured Population Model of Gurtin and MacCamy (1974).

The application of the lemma gives the inequality

$$\int_0^{\Omega} \beta'_0(a)\pi(a)da + \int_0^{\Omega} \beta_0(a)\pi(a)\int_0^{a} \mu'_0(s)ds\,da < 0 \tag{2.8}$$

as a necessary condition for the local asymptotic stability of the equilibrium state of the population model.

Where

$$\pi(a-s) = \exp\left\{-\int_s^{a} \mu_0(\tau)d\tau\right\} \tag{2.9}$$

The integral inequality (2.8) with the application of COA-Sowunmi's lemma involves the two vital parameters of birth and death rates together with their first partial derivatives with respect to the total population. We note that the results from earlier works involve only the partial derivatives. We submit that this result is a stronger condition.

With the inequalities $\beta_0' < \mu_0' < 0$ it evident that the inequality (2.8) holds.

The results obtained in the above analysis imply that

(i) For higher level of population the factors of death will dominate those of birth.

(ii) Factors of death are emigration, intra specific conflicts and competitions especially for space and resources, wars of attrition – survival of the fittest, famine, natural death, disease epidemics and natural and un-natural disasters.

(iii) Factors of birth are immigration and natural reproduction.

Naturally when population is low, no one really cares who gets what. As soon as there is population explosion the factors which cause elimination become prominent. Issues such as sex, tribe, region or state and more dangerously religion are used prominently as factors of elimination when there is population explosion. So our result agrees with reality.

SECTION THREE

MODELLING DISEASE DYNAMICS

3.1 Introduction

In this section we present some of our Mathematical Models of dynamics of diseases with the results of our analysis.

3.2 Mathematical Model of Yellow Fever Disease Dynamics

We present one out of the several models of Yellow Fever Disease Models in this section. We proposed a set of model equations for the dynamics of Yellow Fever disease epidemics; the dynamics involves interactions of the host (human) and the vector (mosquito) populations.

For the purpose of stability analysis, we derived a theorem which will also be highlighted, the set of ordinary differential equations are presented below.

$$\frac{dS}{dt} = \beta_1(S + I + R) - (\mu_1 + \gamma)S - \alpha_1 SM \tag{3.1}$$

$$\frac{dR}{dt} = -\mu_1 R + \gamma S + \alpha I \tag{3.2}$$

$$\frac{dI}{dt} = -(\mu_1 + \alpha + \delta)I + \alpha_1 SM \tag{3.3}$$

$$\frac{dN}{dt} = \beta_2\big(N + (1-\theta)M\big) - \mu_2 N - \alpha_2 IN \quad (3.4)$$

$$\frac{dM}{dt} = \theta\beta_2 M - \mu_2 M + \alpha_2 IN \quad (3.5)$$

With the variables and parameters defined as follow:

$S(t) =$ Susceptible host

$I(t) =$ Infected host

$R(t) =$ Recovered or immuned host

$N(t) =$ Non-virus carrier vector

$M(t) =$ Virus carrier vector

$\beta_1 =$ birth rate of the host

$\beta_2 =$ birth rate of the vector

$\mu_1 =$ death rate of the host

$\mu_2 =$ death rate of the vector

$\alpha_1 =$ contracting rate between S and M compartments

$\alpha_2 =$ contracting rate between N and I compartments

$\alpha =$ recovery rate

$\delta =$ death rate from infection

$\gamma =$ immunization rate

$\theta =$ proportion of M that are infected vertically.

The stability analysis carried out revealed a lower bound on the immunization rate with the inequality

$$\alpha + \delta < \gamma \quad (3.6)$$

We thus posit that the minimum tolerable level of immunization is given by

$$\gamma_{\min} = \alpha + \delta \tag{3.7}$$

for the effective control of Yellow Fever Disease epidemics in a prone community.

In essence, where adequate and accurate statistical data on epidemiology are available and reliable, the proportion of the population that needs to be immunizes or vaccinated in order to avert epidemics outbreak can be estimated using this result.

3.3 Mathematical Model of HIV/AIDS Dynamics

We present here one of the Models on HIV/AIDS, An Infection Age-Structured Mathematical Model of the HIV/AIDS Disease Dynamics. Parameters on the effects of public campaign and the effort at slowing down the death of the victims possibly through drugs administration are introduced.

The model equations are given by:

$$\frac{dS(t)}{dt} = \beta\left[S(t) + R(t)\right] - (\gamma + \mu)S(t) + \theta\beta I(t) - \alpha S(t)I(t) \tag{3.8}$$

$$\frac{dR(t)}{dt} = \gamma S(t) - \mu R(t) \tag{3.9}$$

$$\frac{\partial \rho(t, \tau)}{\partial t} + \frac{\partial \rho(t, \tau)}{\partial \tau} + \left[\mu + \sigma(\tau)\right]\rho(t, \tau) = 0 \tag{3.10}$$

$$\sigma(\tau) = \delta\, e^{-k(T-\tau)} \tag{3.11}$$

$$I(t) = \int_0^T \rho(t, \tau)d\tau \tag{3.12}$$

$$\rho(t, 0) = B(t) = \alpha S(t)I(t) + (1-\theta)\beta I(t), \rho(0, \tau) = \phi(\tau) \tag{3.13}$$

$$S(0) = S_0;\ R(0) = R_0;\ I(0) = I_0 \tag{3.14}$$

with the parameters given by

β = natural birth--rate for the population

μ = natural death--rate for the population

α = rate of contracting the HIV virus

$\sigma(\tau)$ = death rate from infection

δ = maximum death rate from infection

k = measure of the effectiveness of efforts at slowing down the death of infected members.

γ = rate of removal of the susceptibles into the removed class; due to public campaign; i.e. measure of the effectiveness of the public campaign against infection.

θ = the proportion of the offspring of the infected which are virus - free at birth;

$0 \leq \theta \leq 1$.

t = time; τ = infection age.

T = maximum infection age; i.e. when τ = T the infected member dies of the disease.

Our analysis reveals that when k is very low the origin will be stable while the non-zero state will be unstable; and the reverse when k is high.

In Epidemiological term, a low level of k indicates high rate of death among the infected, hence the stability of the origin leading to possible extinction of the population. While a high level of k indicates longer life span for the infected and hence the instability of the origin and stability of the non--zero state. This suggests a usefulness of the use of measures like the anti--retroviral drugs to enhance the life--span of the infected.

We also obtained critical values for k_c and γ_c
Concluding that the public awareness campaign should target a success level not below the value γ_c while the slowing down of infected victim's death should not slide below k_c which may ensure the non-extinction of the population.

3.4 Mathematical Model of Avian (Bird Flu) Dynamics

In this work, we propose a deterministic mathematical model of the dynamics of the Avian Influenza (bird flu) as a set of six ordinary differential equations. The model assumes the interactions of three major players of Migratory and Domesticated (Ground) birds' populations with the Human population. We obtained the characteristic equation and analyzed the equilibrium states for stability in order to gain insight into the dynamics of the flu pandemic.

Avian influenza is an infection caused by avian (bird) influenza (flu) virus. The virus is very contagious and deadly among birds including domesticated birds such as chickens, ducks and turkeys among others. Records of infection of humans are known to be very deadly as infected humans rarely survive the disease.

We define
G_1 = the ground birds population which are susceptible.

G_2 = the ground birds population which are infected.
M_1 = the migratory birds population which are susceptible.
M_2 = the migratory birds population which are infected.
P_1 = the human population which are susceptible.
P_2 = the human population which are infected.

The model equations are given as follows.

$$\frac{dG_1}{dt} = (\beta_1 - \mu_1)G_1 - \alpha_1 G_1 (G_2 + M_2 + P_2) \qquad (3.13)$$

$$\frac{dG_2}{dt} = \alpha_1 G_1 (G_2 + M_2 + P_2) + (\beta_1 - \mu_1 - \delta_1)G_2 \qquad (3.14)$$

$$\frac{dM_1}{dt} = -\alpha_2 M_1 M_2 + (\beta_2 - \mu_2)M_1 \qquad (3.15)$$

$$\frac{dM_2}{dt} = \alpha_2 M_1 M_2 + (\beta_2 - \mu_2 - \delta_2)M_2 \qquad (3.16)$$

$$\frac{dP_1}{dt} = -\alpha_3 P_1 G_2 + (\beta_3 - \mu_3)P_1 \qquad (3.17)$$

$$\frac{dP_2}{dt} = \alpha_3 P_1 G_2 + (\beta_3 - \mu_3 - \delta_3)P_2 \qquad (3.18)$$

With appropriate parameters' definition.

The result of our analysis showed that quite similar to the case when the human population was not included in the interactions, the analysis reveals that once the epidemics is introduced into the populations involved in the dynamics, the tendency for the wiping out of the populations is imminent; since the zero state is stable while the non-zero state is unstable.

This result lays credence to the current control measure whereby infected humans are quarantined while infected birds are wiped out and the environs cordoned and heavily fumigated.

SECTION FOUR

MODELLING SOME ECOLOGICAL DYNAMICS

4.1 Introduction

In this section we present some of our Mathematical Models of some ecological dynamics.

4.2 Mathematical Model of Pollutant in a Flowing River

Beltrami (1989) proposed the model equations for the organic pollutant and the oxygen level in a flowing river. We studied this model, made some improvement and solved the resulting equations to obtain analytical solutions. The solutions give the pollutant and oxygen profiles in a flowing river over time.

The solutions obtained are as follow:

$$P(t) = \frac{1}{\sqrt{4\pi\alpha t}} \exp\left[-\left(\frac{\beta^2 + 4\alpha\gamma}{4\alpha}\right)t\right] \int_0^L e^{-wx} \left\{ \int_{-\infty}^{\infty} e^{-ws} \exp\left[-\frac{(s-x)^2}{4\alpha t}\right] \phi(s) \right\} dx \quad (4.1)$$

This gives the total pollution level in the river at time t

$$Q(t) = q_m L - \frac{\gamma\beta K}{\delta^2}\left[\exp\left(-\frac{\delta L}{\beta^2}\right) - 1\right] - \frac{\gamma}{\delta\sqrt{4\pi\alpha t}} \exp\left[-\left(\frac{\beta^2 + 4\alpha\gamma}{4\alpha}\right)t\right]$$

$$\int_0^L e^{wx} \left\{ \int_{-\infty}^{\infty} e^{-ws} \exp\left[-\left(\frac{(s-x)^2}{4\alpha t}\right)\right] \phi(s) ds \right\} dx \quad (4.2)$$

This gives the total Dissolved Oxygen in the river at time t, in the interval $[0, L]$.

The variables and parameters are defined as:

$P(t) =$ Pollutant level

$Q(t) =$ Dissolved Oxygen level

α = diffusion coefficient

β = advection (flow) velocity

γ = coefficient decomposition of organic matter.

$\delta =$ is the re-oxygenation rate coefficient, the rate at which oxygen is absorbed into the water from the surrounding atmosphere.

$K =$ is the constant rate at which the pollutant is released into the water body

$x =$ is the distance downstream of the point of pollutant discharge;

$t =$ is the time.

The Mathematical Analysis gives:

(i) The critical distance x_c downstream, which represents the point of lowest dissolved oxygen content in the river.

(ii) The lowest (minimum) dissolved oxygen level (content) q_c in the river as a result of the quantum of pollutant.

These estimates can be useful to health and environmental workers.

4.3 Algae Population on ocean surface

Beltrami (1989) proposed models for the algae population on a water body such as the ocean where there could be turbulence and flow. We studied the models, with some modifications and analyzed to obtain some important physical conditions that can ensure the sustenance or annihilation of such water organisms.

Our analysis gave us

$$L_{min} = \frac{r}{v}\left[\frac{4\sqrt{2}}{3}\left[\frac{r}{v}\left(\frac{\rho_1^2}{2} - \frac{\rho_1^3}{3K}\right)\right]^{\frac{3}{2}} - \frac{16\sqrt{2}}{15}\left[\frac{r}{v}\left(\frac{\rho_1^2}{2} - \frac{\rho_1^3}{3K}\right)\right]^{\frac{5}{2}}\right] \qquad (4.3)$$

which gives the minimum length of the patch which has to be maintained for the phytoplankton to still be sustained as a community of organism. It will be observed that the patch length varies directly as the growth rate (r) and inversely as the intensity of diffusion (v). This means that the length of the patch increases with increased growth rate and decreased intensity of diffusion.

$K =$ the environmental carrying capacity.
$r =$ growth or reproduction rate.
$v =$ intensity of diffusion measuring ocean turbulence.
$\rho(x, t) =$ the density of the population at distance x at time t.

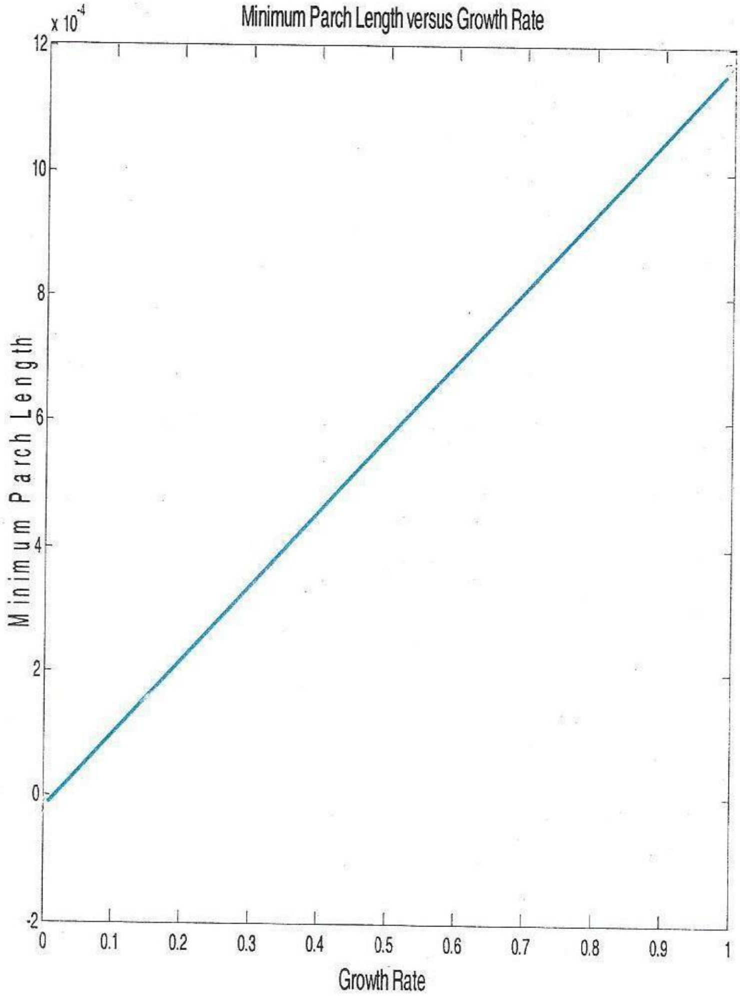


Figure 4.1: Minimum Parch Length versus Growth Rate

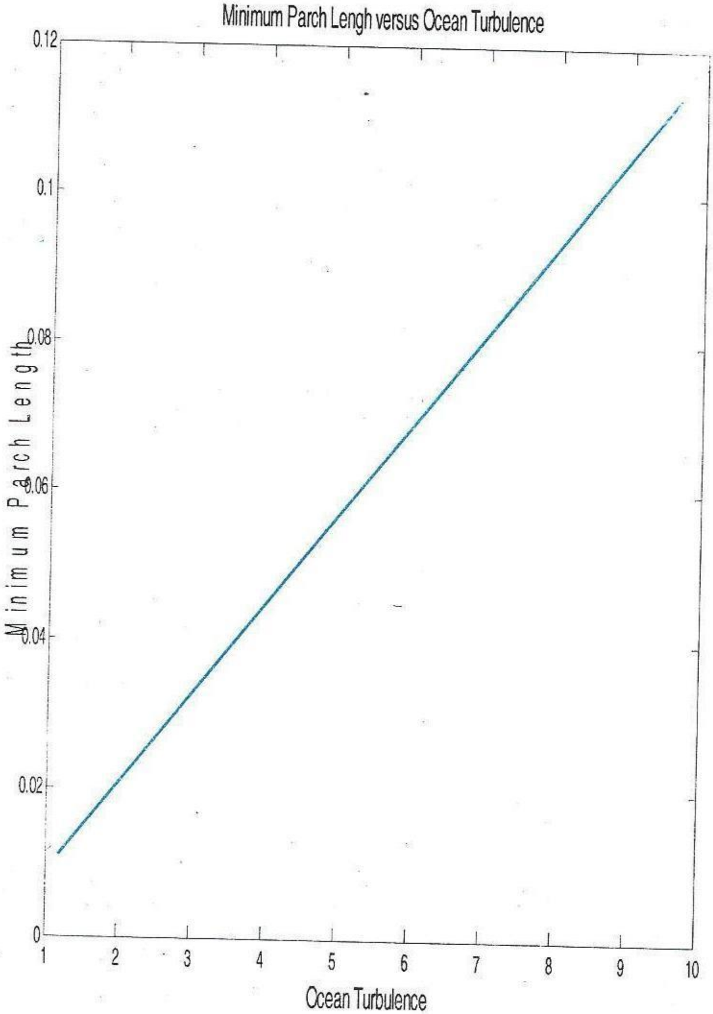


Figure 4.2: Minimum Parch Length versus Ocean Turbulence

SECTION FIVE

ADDENDUM

5.0 Introduction

In this section we take a glimpse into the theatre of Mathematical Modelling of Population and Ecology with two examples.

5.1 Flu in a Boarding School

Suppose there were to be an outbreak of Flu in a closed environment such as a fully boarding school. We model the scenario with the following assumptions.

(i) There are 500 fully resident students in the boarding school and all reported same day in a new term.

(ii) Other staff with their dependants living in the school premises are 100 persons.

(iii) One of the students reported to school with the Flu virus.

(iv) An infected individual is capable of infecting two (2) persons in one day (24 hours).

(v) An infected individual can recover from the Flu within 5 days.

(vi) The Flu is contagious but not fatal, i.e. does not cause death.

(vii) An infected person is not sent home but rather is taken through a recovery process.

The S-I-R model is adopted.

At time $t = 0$, $S_0 = 599$, $I_0 = 1$, $R_0 = 0$. The model equations are:

$$\frac{dS}{dt} = -\alpha SI \tag{5.1}$$

$$\frac{dI}{dt} = \alpha SI - \gamma I \tag{5.2}$$

$$\frac{dR}{dt} = \gamma I \tag{5.3}$$

Computation of Parameters

1. An infected person can recover within 5 days, so we take the recovery rate to be $\gamma = \frac{1}{5} = 0.2$.
2. An infected person can infect 2 persons in a day, so we take the infection rate to be $\alpha = \frac{2}{600} = 0.003$.

Analysis

At equilibrium state, equation (5.2) gives

$I = 0$ or $S = \frac{\gamma}{\alpha} = \frac{0.2}{0.003} \approx 67$.

We note that $\frac{dI}{dt} < 0$ when $S < 67$, the Flu gradually eased out; and $\frac{dI}{dt} > 0$ when $S > 67$, indicating that the Flu worsens. So we conclude that a safe immunization level against the Flu for the closed population will be 600 – 67 = 533.

5.2 Hetero and Homo Sexuality

In this theatre, we use the logistic population model highlighted in section one to illustrate the possible scenario if the world chooses to continue with the God given and ordained Heterosexuality in which male and female cohabit to reproduce or homosexuality in which single sex cohabit.

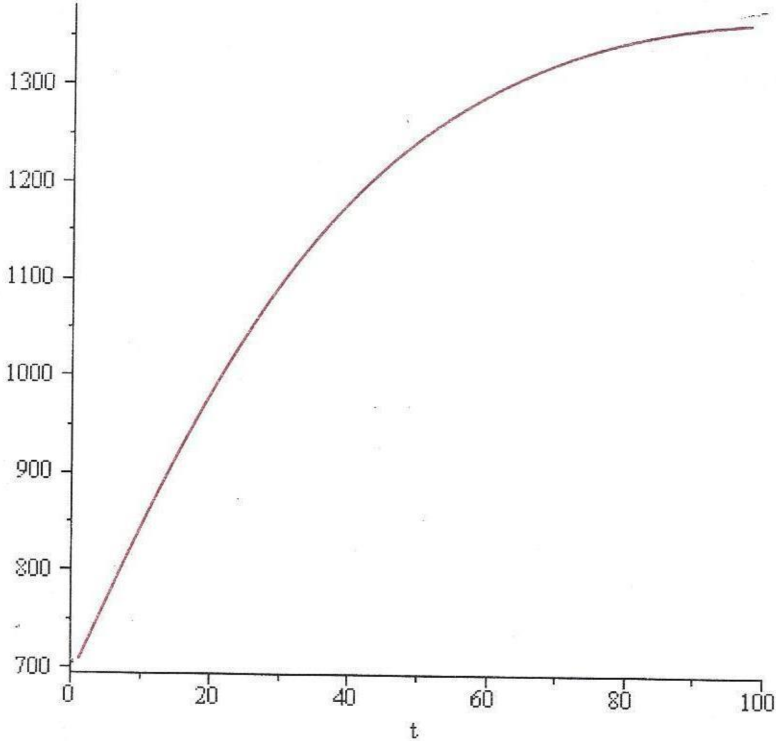
The environmental carrying capacity remains K, that is the maximum population which the environment can sustain. The model equation is reproduced below.

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right); \qquad P(0) = P_0 \tag{5.3}$$

The analytical solution of (5.3) is

$$P(t) = \frac{K\,P_0}{\left(K - P_0\right)e^{-rt} + P_0} \tag{5.4}$$

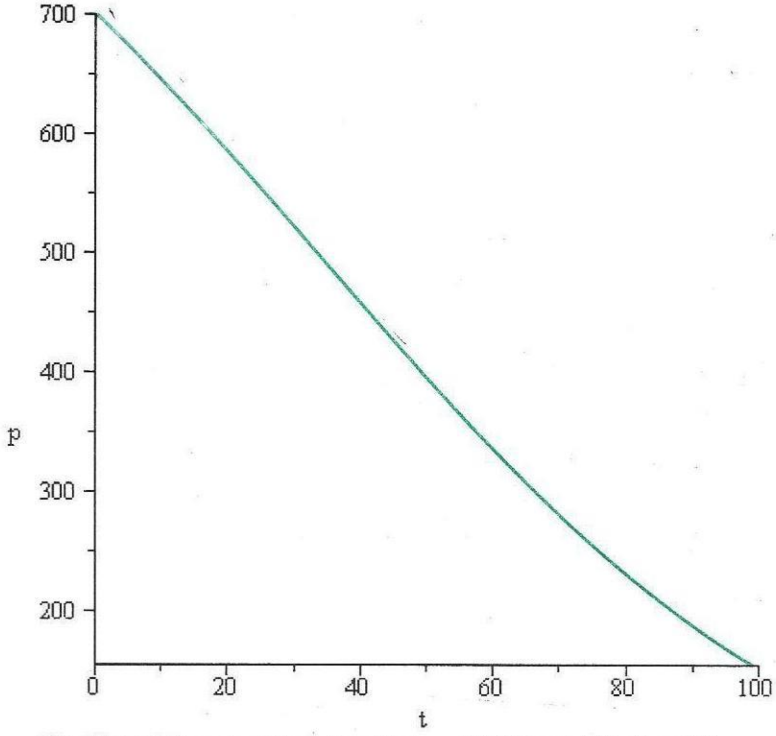
In a Heterosexual world $r > 0$ as there is sustaining reproduction and the graphical representation of the logistic model is given below in figure (5.1) below.



The Plot of P against time (t) with r = 0.043, k = 1,400, P_0 = 700

Figure 5.1; $r > 0$

On the other hand in a Homosexual world $r < 0$ as there is no sustaining reproduction and the graphical profile of the logistic model is given below in figure (5.2) showing that the population will face definite extinction.



The Plot of P against time (t) with r = − 0.025, k = 1,400, P_0 = 700

Figure 5.2; $r < 0$

As can be seen in this graph, the world population will go into extinction in 100 years time if the world embraces homosexuality, it is a danger signpost. Hope the advocate can have a rethink taking cognizance that the command of God is that mankind should be fruitful and multiply, Genesis 1:28.

CONCLUDING REMARKS

The practical importance of understanding the dynamics of infectious diseases has steadily increased over the years. Apart from wars, famine, accidents and natural disasters, other potent mortality factor for the human race is through infectious diseases.

There has been a long struggle against infectious diseases with outstanding accomplishment in medicine recent WHO reports indicates that the battle is still not over as new scenario keep emerging. For examples,

17 million people died from infectious diseases in 1995.

Every year, hundreds of millions suffer from malaria, and over one million children lose their lives to the disease. A major obstacle to the control of malaria is the emergence of drug resistant strains.

In the Nigerian Daily Sun Newspaper of Thursday April 10, 2014 is a caption "Ebola, real threat to Nigeria – FG." According to this news item there had been 137 reported cases with 100 deaths. This implies a high mortality rate of $100/137 = 0.7$, in the West African sub-region.

One of the very important questions in the mathematical studies of epidemics is the possibility of the eradication of the disease. When a model is formulated, the mathematician, in order to answer the above questions, can study the disease free or endemic equilibrium states for stability. If the disease free equilibrium exists and is stable, the conclusion drawn is that the disease will steadily wear out in the population. On the other hand if it is the endemic state that is stable this is an indication that the disease could persist in the population.

In the formulation of such models, parameters are introduced depicting drug administration and effectiveness, immunization, outright quarantine. The results of the mathematical analysis are then interpreted relating to the significant or vital parameters.

In addition to the control of epidemics, this field of study could also lead to useful policy formulation for bio-diversity problems and the conservation of endangered species. This brings to the fore, the importance of collaborations among mathematicians, biologists, health and environmental scientists and other related disciplines.

ACKNOWLEGEMENT

Psalm 40:1-5, I waited patiently for the Lord; ...... He set my feet upon a rock, and established my goings. I give all glory, honour and praise to my Redeemer and Saviour, the Lord Jesus Christ. He is good to me.

I am deeply grateful to my mother, Late Awawu Akanke Akinwande, she spent her meager resources to give me good educational foundation. She had enough to build a house but she chose to train me instead, may her soul rest in perfect peace; how I wish she were to be here today, but alas "it is appointed onto men once to die, but after this the judgement" - Hebrews 9:27.

My teachers and academic mentors, Late Professors C. O. A. Sowunmi, H. O. Tejumola, G. O. Olaofe then at the University of Ibadan. And others who are still alive and professing, Professors Olabisi Ugbebor, G. O. S. Ekhaguere, Olusola Akinyele, V. A. Babalola, Aderemi Kuku, all these in notable ways contributed to my academic advancement, I am grateful to them.

I thank the Director and Chief Executive of the National Mathematical Centre, Abuja, Professor A. R. T. Solarin, for writing the Foreword for my book in memory of Professor Sowunmi and for gracing this occasion, he has also imparted positively on my academic accomplishment.

I wish to express my gratitude to this special set of colleagues, All the Deans of School of Science & Science Education during my tenure as Head of Department, Prof (Mrs.) H. O. Akanya, Prof G. D. Momoh and Prof M. Galadima. Members of my Department, Prof K. R. Adeboye, Prof Y. M. Aiyesimi, Dr. Y. A. Yahaya, Dr. U. Y. Abubakar, Dr. Isah Audu, Dr. M. Jiya, Dr. A. Mohammed, Dr. D. Hakimi, Dr. R. O. Olayiwola, Dr. A. Enagi, Dr. A. Ndanusa, Dr. G. Bolarin, Dr. Sirajo Abdulrahman, Dr. D. Shehu, Dr. (Mrs.) A. T. Cole.

I wish to thank the Chairperson of the University Colloquium and Seminars Committee, Prof (Mrs.) Z. D. Osunde and all the members of her committee for a job well done as always.

I wish to commend the members of the editorial team for this inaugural, Dr. Sirajo Abdulrahman, Dr. A. Enagi, Dr. D. Hakimi, Mr. Samuel Abubakar and Mrs. Temitope Oluwafemi.

I wish to thank His Royal Highness, Alhaji (Dr.) Umaru Faruq Bahago, OON, Emir of Minna. I wish to place on record your love and care for all the people in your Emirate regardless of sentiments, may your reign continue to enjoy the peace and blessings of God, amen.

I thank the Vice-Chancellor, Professor Musbau A. Akanji; he rescheduled the date of this inaugural lecture so that he can be present to chair it, I quite appreciate that sir, I pray that God will make your tenure to witness outstanding accomplishments.

My gratitude goes also to Pastor and Pastor (Mrs.) Jimi Ajibade, the Pastor In Charge of the Redeemed Christian Church of God, Niger Province I, Minna and other Pastors here present.

I am grateful to my sister, Mrs. Aderemi Obadara, she actually counseled our mother to make my education of topmost priority, thank you ma. My mother in law too is here at 86, Mrs. Comfort Abike Olurebi. Also my nieces, Mr. & Mrs. Abiodun Toriola, Michael Olurebi, Gbemisola Akinwande and my son in law, Nathaniel Niyi Oluwafemi.

Finally, I thank my wife, Arinola Akinwande, for she forsook her rosy job at the University of Ibadan to come to Minna, taking care of me and the children; Temitope (now Mrs. Oluwafemi), Tobiloba and Temitayo. I feel complete with you beside me, may God continue to keep you for me, amen.

Thank you all.

REFERENCES

[1] Abdulrahman S., Akinwande, N. I., Awojoyogbe O. B. & Abubakar U. Y. Mathematical Analysis of the Control of Hepatitis B Virus in a Population with Vital Dynamics. *The Pacific Journal of Science & Technology*. 14(1), 188 – 202, (2013).

[2] Abdulrahman, S., Akinwande, N. I., Awojoyogbe, O. B. & Abubakar, U. Y. Effects of Condom Usage and Vaccination on the Transmission dynamics of Hepatitis B Virus Infection. *Journal of the Nigerian Association of Mathematical Physics,* 24, 249 -256, (2013).

[3] Abdulrahman, S., Akinwande, N. I., Awojoyogbe, O. B. & Abubakar U. Y. Mathematical Solutions for Hepatitis B Virus Infection in Nigeria. *Journal of Research in National Development.* 11(1), 302-313, (2013).

[4] Abubakar S, Akinwande, N. I. & Abdulrahman S. A Mathematical Model- of Measles Disease Dynamics; J *Sc. Tech. & Math. Ed. (JOSTMED)* F.U.T., Minna. 8(2), 144 - 152, (2012).

[5] Abubakar, S., Akinwande, N. I., Abdulrahman, S. & Oguntolu F. A. Bifurcation Analysis on the Mathematical Model of Measles Disease Dynamics. *Universal Journal of Applied Mathematics.* 1(4), 212-216, (2013).

[6] Abubakar, S., Akinwande, N. I., Jimoh, O. R.,Oguntolu, F. A. & Ogwumu, O. D. Approximate Solution of SIR Infectious Disease Model Using Homotopy Pertubation Method

(HPM). *Pacific Journal of Science and Technology* (PJST), 14(2), 163-167 (2013).

[7] Akinwande N. I., Mohammed A. A. & Jiya M. A. Mathematical Model of Competition in two interacting Population; Proceedings of the Annual National Conference of the Mathematical Association of Nigeria, (MAN) pp 15–20, (2006).

[8] Akinwande N. I. On the Application of Differential Equations in the Mathematical Modelling Population Dynamics; Annual School of Science & Science Education Conference, F.U.T. Minna, pp 77 – 84, (2006).

[9] Akinwande N. I. Saturation Process in Prey-Predator Population Interaction; Book of Readings of the First Annual School of Science & Science Education Conference; F.U.T. Minna, pp 14 – 18, (2005).

[10] Akinwande, N. I. & Abdulrahman S. A. Mathematical Model of Pollutant Decomposition via Oxygenation in a Flowing River; *Journal of Pure & Applied Sciences, ATBU, (Science Forum)* 12(1), 19 – 21, (2008).

[11] Akinwande, N. I. & Ugbebor O. O. Analytical Geometry and Mechanics; Publisher: Associated Books Maker, Ibadan, Nigeria; 109 pgs; ISBN: 978 - 345 - 328 - 9; Chapters contributed: Chapters 1, 2 & 3, (2000).

[12] Akinwande, N. I. A. Mathematical Model of the Chaotic Dynamics of the AIDS Disease Pandemic; *J. Nig. Math. Soc.* 24: 8 – 16; (2005).

[13] Akinwande, N. I. An Infection Age--Structured Mathematical Model of the Dynamics of HIV/AIDS Disease Dynamics; *Nig. J. of Sc. Research (NJSR), Ahmadu Bello University, Zaria*. 5(1), 50 -54, (2005).

[14] Akinwande, N. I. Introduction to Fourier Series and Transform; Publisher: Associated Books Maker, Ibadan, Nigeria; 50 pgs; ISBN – 978 - 265 - 932 – 0, (2004).

[15] Akinwande, N. I. A. Deterministic Mathematical Model of the Avian Influenza (Bird Flu) Dynamics; *Science Focus, Ladoke Akintola University of Technology, Ogbomoso*. 12(1), 19 – 21, (2007).

[16] Akinwande, N. I. A. Mathematical Model of the Avian Influenza (Bird Flu) Dynamics Involving Human Interactions; *Science Focus, Ladoke Akintola University of Technology, Ogbomoso.* 12(1), 129 – 132, (2007).

[17] Akinwande, N. I. A. Mathematical Model of the Dynamics of the HIV/AIDS Disease Pandemic; *J. Nig. Math. Soc.* 25, 99 – 108, (2006).

[18] Akinwande, N. I. A. Mathematical Model of Yellow Fever Epidemics; *Afrika Matematikaserie.* 6(3), 49 – 59, (1996).

[19] Akinwande, N. I. A. Time Discrete Mathematical Model of Yellow Fever Disease Dynamics; *The Universitas, Journal of the University of Ghana, Legon*. 12, (1999).

[20] Akinwande, N. I. Application of Laplace Transform to the Stability Analysis of the non-zero Equilibrium State of a Yellow Fever Disease Dynamics Model; *Science Focus, Ladoke Akintola University of Technology, Ogbomoso.* 10(2), 166 – 176, (2005).

[21] Akinwande, N. I. Application of Sowunmi's Proposition on the Study of the Characteristic Equation of a Dynamical System: *Advances in Mathematics, Proceedings of a Memorial Conference in Honour of Late Professor C.O.A Sowunmi, Department of Mathematics, University Of Ibadan, Ibadan, Nigeria.* 1, 14 – 19; (2009).

[22] Akinwande, N. I. COS-Sowunmi's Lemma – A Result on The Stability Analysis of the Equilibrium States of Mathematical Models of Population Dynamics. Publisher: Darkol Press and Publishers, Lagos, Nigeria; 8 pgs; ISBN – 978 - 978 – 936 – 921- 8, (2013).

[23] Akinwande, N. I. Introduction to Tensor Analysis; Publisher: Associated Books Maker, Ibadan, Nigeria; 70 pgs; ISBN: 978 – 265 – 939 – 8, (2000).

[24] Akinwande, N. I. Local Stability Analysis of Equilibrium State of a Mathematical Model of Yellow Fever Epidemics; *J. Nig. Math. Soc.* 14, 73 – 79; (1995).

[25] Akinwande, N. I. Mathematical Methods I - MAT 341; Publisher: Center for External Studies, University of Ibadan, Ibadan, Nigeria. 124 pgs; ISBN: 978 – 021 – 140 – 3, (1997).

[26] Akinwande, N. I.; Ogunfiditimi F. et al. College Algebra and Trigonometry; Publisher: Associated Books Maker, Ibadan, Nigeria; 181 pgs; ISBN - 978 - 2659 - 36 - 3; Chapters contributed: Chapters 2, 5 & 6, (2005).

[27] Akinwande, N. I. On the Analysis of a Characteristic Equation Arising from the Stability Analysis of Steady State of some Dynamical Systems; *J. Nig. Math. S Soc.* 28, 169 – 179, (2009).

[28] Akinwande, N. I. On the Boundedness of the Host Population in an Age- Structured Model of Yellow Fever Epidemics; *J. Agric. Sc. & Tech, Univ. of Agric, Makurdi.* 12(2), 117 – 124, (2003).

[29] Akinwande, N. I. On the Boundedness of the Vectors' Population in an Age-Structured Mathematical Model of Yellow Fever Epidemics; Proceeding of the Directions in Mathematics Conference in honour of Prof. H.O. Tejumola, University of Ibadan, (1997).

[30] Akinwande, N. I. On the Characteristics Equation of a Non-Linear Age-Structured Population Model; *ICTP, Trieste, Italy Preprint* IC/99/153; (1999).

[31] Akinwande, N. I. On the Characteristics Equation of a Non-Linear Age- Structured Population Model. *J Sc. Tech. & Math. Ed. (JOSTMED) F.U.T., Minna.* 6(1), 1; (2003).

[32] Akinwande, N.I. Mathematical Modelling of Population Dynamics, Lecture Note presented at the *11th Regional*

College on Modelling, Simulation and Optimization, University of Cape Coast, Ghana, (2009).

[33] Akinwande, N. I. On the Analysis of a Characteristic equation Arising from the Stability Analysis of Steady States of some Dynamical Systems; *J. Nig. Math. Soc.* 28, 1 – 11, (2009).

[34] Akinwande, N. I., Aiyesimi Y. M. et al. College Differential and Integral Calculus; Publisher: Associated Books Maker, Ibadan, Nigeria; 195 pgs; ISBN – 978 – 2659 – 42 - 8; Chapters contributed: Chapters 8, 9 & 10, (2006).

[35] Anderson, R. M. & May, R. M. Infectious Diseases of Humans. Oxford University Press, Oxford U.K. (1991).

[36] Apostol, T. M. *Mathematical Analysis; A Modern Approach to Advanced Calculus*; Addison-Wesley, London. (1964).

[37] Bellman, R. & Cooke, K. L. *Differential Difference Equations*; Academic Press, London; (1963).

[38] Beltrami, E. *Mathematics for Dynamics Modelling*; Academic Press Inc London (1989).

[39] Benyah, F. Introduction to Mathematical Modeling; *7th Regional College on Modeling, Simulation and Optimization, University of Cape Coast, Ghana.* (2009).

[40] Chiu, C. A. Numerical Method for Non-linear Age Dependent Population Models, *J. Diff. and Int. Eq*. 769 - 782; (1990).

[41] Churchilll R. V. *Fourier series and Boundary Value Problems*; McGraw Hill; (1963).

[42] Goel N. S, Maitra S. C. & Montroll E. W. *On the Volterra and Other Non-Linear Models of Interacting Populations*; Academic Press, N.Y, London; (1971).

[43] Gurtin M. E. & MacCamy R. C. Non-Linear Age-Dependent Population Dynamics; Arch. Rat. Mech. Anal. 54, 281 - 300; (1974).

[44] Kermark, W. O. & MeKendrick, A. G. A Contribution to the Mathematical Theory of Epidemics. Proc. Roy. Soc. A 115, 700 – 721. (1927).

[45] Jack H. *Theory of Functional Differential Equations*; Springer Verlag, New York, Heidelberg, Berlin; 337 - 341. (1977).

[46] Jumping S. Partial Differential Equation and Mathematical Biology; *Mathematical Biology Journals*, College of William and Mary Library and Network, Spring; (2004).

[47] Reju C. O., Reju S. A. & Akinwande, N. I. Optimal Control of Linear Order Non_Dispersive Waves. *AMO – Advanced Modeling and Optimization*, 14(3), 471 – 480, (2012).

[48] Akinwande, N. I., Aiyesimi Y. M. et al. College Differential and Integral Calculus; Publisher: Associated Books Maker, Ibadan, Nigeria; 195 pgs; ISBN – 978 – 2659 – 42 - 8; Chapters contributed: Chapters 8, 9 & 10, (2006).

[49] Shigesada, N. & Kawasaki, K. Biological Invasions: Theory and Practice. Oxford University Press, Oxford. (1997).

[50] Sowunmi, C. O. A. On a Set of Sufficient Conditions for the Exponential Asymptotic Stability of Equilibrium States of a Female Dominant Model; *J. Nig. Math Soc.* 6, 59 – 69; (1987).

[51] Takeuchi, Y., Iwasa, Y. & Sato, K. Mathematics for Life Science and Medicine. ISBN-13 978-3-540-34425-4. Springer Berlin Heidelberg New York. (2006).