



**FEDERAL UNIVERSITY OF TECHNOLOGY
MINNA**

**PREDICTIVE STOCHASTIC
MODELING OF LEPROSY DISEASE,
HUMAN HEALTH CONDITION AND
DESERTIFICATION IN NIGERIA:
THE UNTAPPED OPEN
SECRET INGREDIENTS**

By

PROF. USMAN YUSUF ABUBAKAR
B.Ed (Zaria), M.Sc (Jos), PGD, PhD (Minna)
Professor of Mathematics

INAUGURAL LECTURE SERIES 57

26TH OCTOBER, 2017



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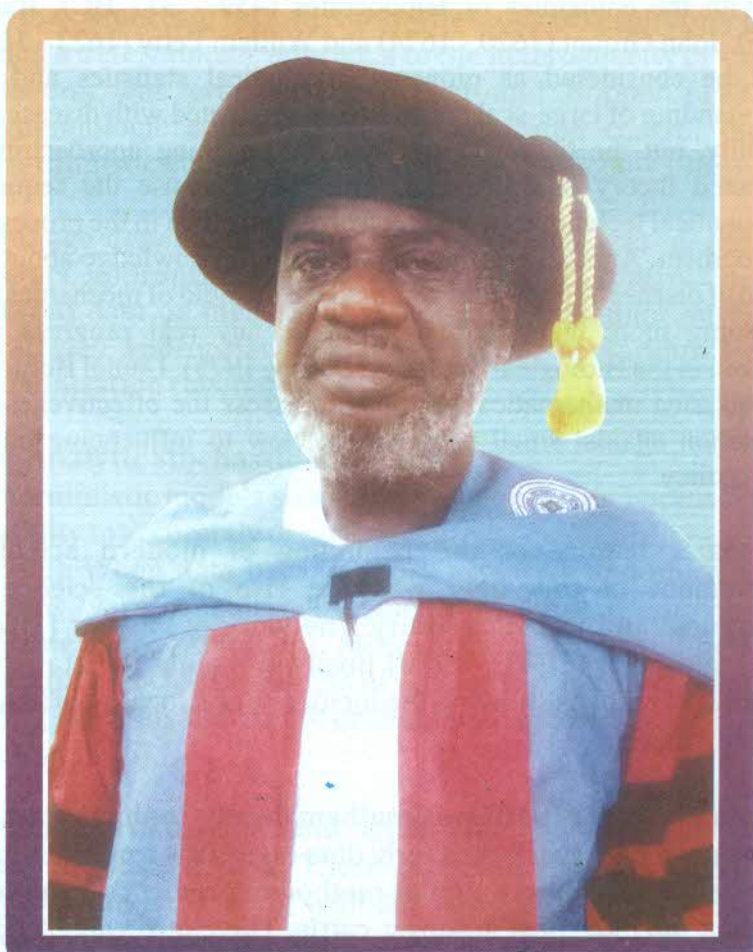
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1. The Beginnings

The modeling of disease started as far back as the ancient Greeks, with the epidemics of Hippocrates (459 - 377 BC), Bailey (1975). John Grount (1620 - 1674) and William Petty (1623 - 1687) could be considered as pioneers of medical statistics and the understanding of large-scale phenomena connected with disease and mortality, but the time was not ripe for anything approaching a connected theory of epidemics. This was because the requisite mathematical techniques were themselves only then in the process of development. Another reason was insufficient knowledge about the spread of disease. A good start was made in the field of mechanics and astronomy more than 200 years before any real progress was achieved in the Biological Sciences (Barley, 1975). Daniel Bernoulli in 1760 used mathematical methods to assess the effectiveness of inoculation against small pox, with a view to influencing public health policy.

The major feature of the beginning of modern scientific achievement in this field was the rise of the science of bacteriology in the 19th century. The work of Pasteur (1822 - 1895) and Koch (1843 - 1910) involved mainly the statistical appraisal of records showing the incidence and locality of known cases of disease.

The work of Farr (1840) was mathematically sophisticated. He fitted a normal curve to quarterly data on deaths from smallpox. Brownlee (1866) used a similar method to predict the course of outbreak of rinderpest amongst cattle. The curve was fitted to four rising successive monthly totals and extrapolated values used for prediction. Although observed and predicted curves were both bell-shaped, agreement in detail was not very good.

The work of Farr and Brownlee involved more of curve fitting and prediction.

2. Mathematical Modelling

Ross (1911) developed a mathematical model for malaria, which attempted to take into account a set of measures describing various aspects of transmission. The study of respiratory disease using a deterministic approach to the heterogeneity of spread of infection was provided by Becker and Hopper (1983). An epidemiologic application of sophisticated control theoretic deterministic modeling was provided by Hethcote (1983).

The age-dependent immunization model was designed to predict appropriate strategies for disease control. Hethcote utilized data on measles and rubella to determine vaccination strategies appropriate for their control at various levels of immunization coverage.

2a. Analytic Stochastic Modeling

Deterministic models soon lost their popularity because of their inability to accurately describe recurrent cycles of disease, Bailey (1982). When data became more extensive and much smaller groups were considered, elements of "chance and variation" became more prominent. Mckendrick (1926) was the first to construct stochastic models of epidemic processes. Greenwood (1931) gave an alternative probability treatment five years later, Bailey (1975).

"Continuous infection" and "chain binomial" stochastic models were introduced next. These probability models were more appropriate for dealing with smaller groups in which random variation would play a larger role. Although these models achieved popularity they are usually mathematically and computationally more complex than the simple deterministic models, Korve (1993).

Stochastic models now appear more frequently in the study of diseases, Bailey (1975). Kimber and Crowder (1984) proposed a

model to analyze resistance times to infection under treatment. A general stochastic model was proposed by Hillis (1979).

Several stochastic models have been presented to describe distributions of infectious disease over time and space, including Korve (1993) and Goldacre (1977) who attempted an analysis of meningitis using space-time clustering techniques introduced by Knox (1964) to detect the existence of factors associated with infection.

3. Modeling Approaches for Disease Control

Generally, there are three modeling approaches for disease control: Deterministic, Analytical stochastic, and Simulation, usually stochastic.

Deterministic and stochastic models were developed in the early part of the 20th century. The etiology of disease is of primary concern to many epidemiologist and can be seen either in a deterministic or stochastic framework.

A deterministic perspective is one in which factor x cause y if (all other factors being held constant) a change in the value of x results in a change in the values of y , in a completely prescribed way tracing out a mathematical function of some form.

In practice, probability theory and statistical techniques are used to assess evidence regarding causality. In any causal analysis of data, the goal is to account for variation in the dependent variable.

Several models of this sort have been utilized to analyze data in studies of infectious diseases, including most commonly linear regression, log-linear analysis, logistic regression, discriminant analysis, and proportional hazards modeling, Korve(1993).

4. Probability

The approach to probability used in this work is the axiomatic, in which probabilities are defined as "mathematical objects" which behave according to certain well-defined rules. Then any other probability concepts, or interpretations, can be used, so long as it is consistent with these rules. Thus probabilities are values of a set function, also called a probability measure; this function assigns real numbers to the various subsets of the sample space. John et al (1980).

5. Random Variables

It frequently occurs that in performing an experiment, we are mainly interested in some function of the outcome as opposed to the actual outcome itself. For instance, in tossing dice we are often interested in the sum of the two dice and are not really concerned about the actual outcome. That is, we may be interested in knowing that the sum is three and not be concerned over whether the actual outcome was (1,2) or (2,1). These quantities of interest or formally, these real-valued functions defined on the sample space are known as random variables. Since the value of a random variable is determined by the outcome of an experiment, we may assign probabilities to the possible values of the random variable.

6. Discrete Random Variable

A random variable that can take on at most a countable number of possible values is said to be discrete. For a discrete random variable X , we define the probability mass function

$$p(a) = P\{X=a\}$$

The probability mass function $P(a)$ is positive for at most a countable number of values of a . That is, if X must assume one of the values x_1, x_2, \dots , then

$$p(x_i) > 0, \quad i = 1, 2, \dots$$

$$p(x) = 0, \quad \text{all other values of } x$$

Since X must take on one of the values x_i , we have

$$\sum_{i=1}^{\infty} p(x_i) = 1$$

7. Continuous Random Variables

A random variable whose set of possible values is uncountable. Let X be such random variable. X is a continuous random variable if there exists a non-negative function $f(x)$, defined for all real $x \in (-\infty, \infty)$, having the property that for any set B of real numbers

$$P\{X \in B\} = \int_B f(x) dx \quad (1)$$

The function $f(x)$ is called the probability density function of the random variable X .

In words, equation (1) states that the probability that X will be in B may be obtained by integrating the probability density function over the set B . Since X must assume some value, $f(x)$ must satisfy

$$1 = P\{X \in (-\infty, \infty)\} = \int_{-\infty}^{\infty} f(x) dx$$

All probability statements about X can be answered in terms of $f(x)$. For instance, letting

$$B=[a,b], \text{ we obtain } P\{a \leq X \leq b\} = \int_a^b f(x) dx$$

8. Stochastic Processes

The family of random variables $X(t)$, $t \geq 0$ indexed by the time parameter t . The values assumed by the process are called 'states' and the set of possible values is called the state space. The set of possible values of the indexing parameter is called the

'parameter space' which can be either continuous or discrete. In the discrete case, the process is represented as $X_n, n = 0, 1, 2, \dots$

The stochastic process occurring in most real-life situations are such that for a discrete set of parameters $t_1, t_2, \dots, t_n, t, T$, the random variables $X(t_1), X(t_2), \dots, X(t_n)$ exhibit some sort of dependence. The simplest type of dependence is the first-order dependence underlying the stochastic process. This is called Markov dependence.

Depending on the nature of the state space and the parameter space, we can divide Markov processes into four classes, which are given here in the form of a table. Wherever the parameter and state spaces are discrete the Markov process is called Markov chains. Otherwise the process is simply referred to as a Markov process or memoryless property.

Table 1: Classification of Markov process

PARAMETER SPACE	STATE SPACE	
	Discrete	Continuous
Discrete	Markov Chain	Markov Process
Continuous	Markov Process	Markov Process

The other typical Markov processes include Semi'Markov, Hidden Markov, Markov Decision and partially Observable Markov processes.

9. Markov Chains

Markov chains is the Markov process with discrete time and parameter spaces whose state space could be finite or countably infinite.

Let $\{X_n, n = 0, 1, 2, \dots\}$ be a Markov chain with a state space $S \subseteq Y = \{0, 1, 2, \dots\}$. While discussing a finite m -state chain, we shall

